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Estimating Potential Output at the Central Bank of Armenia

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Abstract

Potential output and output gap are important concepts in many areas of economic policy. However they are unobservable and can only be measured with some uncertainty. The paper documents some estimation methods used at the Central Bank of Armenia, discusses some relevant criteria for evaluation of the methods and shows the relative performance of the models across a wide range of criteria.

JEL classification: E32, C32.

Keywords: Potential Output, Output Gap, Multivariate Filter, Production Function Aproach

The views expressed in this paper are those of the author and do not necessarily represent the views of the Central Bank of Armenia.

1. Introduction

Economists have tendency to talk about the same thing using different notations and to talk about different things using the same notation. The most striking examples of this phenomenon are the concepts of output gap and potential output. The latter notions are crucial in many areas of economic policy and are used very extensively among the policymakers. However they are unobservable and can only be estimated using some techniques. Theoretically there is no unique economic definition of what is potential output and there can be infinite number of possibilities to decompose the actual output to trend and cycle components. Therefore based on the needs of the policymakers different methods of estimating potential output and output gap have emerged recently.

The concepts of potential output vary from more traditional definitions, such as relating the supply capacity of the economy that can be sustained without creating inflationary and deflationary pressures, to more complex definitions, such that account for financial imbalances etc.

Orphanides and Van Norden demonstrate the unreliability of output gap estimates in real time using a wide range of estimation methods (Orphanides and Van Norden 2002). In other paper Orphanides demonstrates that some important policy mistakes are done because of the misperception about the economy‘s productive capacity and long-run growth (Orphanides, 2003). This points motivate the importance of accurately assessing the output gap and long-run growth in real time.

This paper examines two methods that are employed for estimating potential output and output gap at the Central bank of Armenia. The key focus is on their relative performance compared to the widely used methods, cons and pros of each model and their use in policy making process. First method is a multivariate unobserved components model similar to the one proposed by Ali Alichi et al (2017) and the second method is a simpler version of production function approach, which is used by European Commission (Havik et al (2014)).

The rest of the papers is as follows. Section two presents brief overview of the widely used methods for estimating potential output. Section tree discusses the criteria for evaluating different methods. Section four and five document the two methods used at the central bank of Armenia in details. Section six gives an overview of the data and estimation method. Section seven demonstrates the relative performance of the methods across a wide range of criteria. Section eight concludes.

2. Brief overview of the estimation methods

It should be understood that the concept of output gap is meaningful only when properly defined, before being embedded in a model. Therefore it is useful to discuss some of the common concepts before going to estimation methods. The "traditional" concept of the potential output is the one suggested by the European Commission (Havik et al 2014)

Over the **short run**, the physical productive capacity of an economy may be regarded as being quasi fixed and its comparison with the effective / actual output developments (i.e.in output gap analysis) shows by how much total demand can develop during that short period without inducing supply constraints and inflationary pressures.

Over the **medium term**, the expansion of domestic demand when it is supported by a strong upturn in the amount of productive investment may endogenously generate the productive output capacity needed for its own support. The latter is all the more likely to occur when profitability is high and is supported by an adequate wage evolution with respect to labour productivity.

Finally, over the **long run**, the notion of full employment potential output is linked more to the future evolution of technical progress (or total factor productivity) and to the likely growth rate of labour potential.

A recent modification of the concepts includes the so called "Finance-neutral" potential output, which accounts for financial imbalances and is more linked to sustainable output rather than providing information about the inflationary pressures.

The DSGE literature introduces some recent concepts of potential output: the natural output or flexible price output, which would prevail under the absence of nominal rigidities and the efficient output, which would prevail both under the fully flexible prices and under the perfect competition.

In this paper we consider only the short run definition of potential output which is still dominant concept among the monetary policy makers.

Different approaches to estimate the potential output have emerged recently. One of the most prevalent methods of estimating the potential output and output gaps is the use of univariate statistical filters, such as HP filter. The approach is very simple and requires only the data of GDP, however the simplicity comes with drawbacks. It is well documented in the literature that the HP filter suffers from the end point problem i.e. as the new data becomes available the estimates can be revised significantly. Actually the end point problem is a typical problem for all two-sided filters which use future data to estimate the current position of the economy and there is no practical solution to eliminate this problem completely, nonetheless some methods suffer less and some methods suffer more from this

problem. One practical approach is to extent the sample with forecasts, but the estimated potential is going to be influenced by the forecasts, while the opposite is preferable, meaning that the forecasts should be based on the projected potential. Besides end sample problem the underlying assumptions used by HP filter can be inconsistent with the data. For example if the output gap is very persistent then the white noise assumption of the HP filter may not generate plausible business cycle measures. For more on the drawbacks of HP filter see Hamilton (2017).

Much work has been done to develop multivariate filters for estimating potential output and output gaps (see Laxton et al. (1992), Benes et al. (2010), Blagrave el al. (2015), Alichi et al. (2017), Alichi et al. (2018)). The benefit of these multivariate filters is that the equations of the model are pinned down by economic relationships, such as the Phillips curve which shows the relationship of inflation and output gap, the Okun law which relates the unemployment gap to the output gap etc. So more identifying information allows to estimate the cycle with greater accuracy and makes the estimates more consistent with the traditional definitions of potential output which motivates such pervasive usage of multivariate filters among the monetary policy makers. The approach is very flexible and easy to replicate the underlying beliefs about the statistical features of variables that are used. Besides this approach can help partially solve the end-sample problem which is prevalent in univariate methods. Alichi et all (2017) showed that the real-time estimates from their approach are more accurate than estimates constructed from naive univariate statistical filters. However the value added of this approach depends on the plausibility of the economic structure imposed in the models.

Another common technique of estimating potential output is the so called production function approach, which decomposes the output to production inputs and considers them separately. For example in case of a Cobb-Douglas production function the output will be a combination of labor, capital and productivity inputs and the potential output will be based on the trend assumptions of the latter variables. In this context the production function approach is not itself a separate methodology but its a combination of underlying methods that are used for estimating the trends of separate components. For example if the trend inputs of production are estimated using the HP filter the estimated potential output and output gap will be subject to the same drawbacks as the HP filtered GDP itself. However the benefit of the Production function approach is that underlying economic structure of the function can be used to discuss the components separately and make projections based on the likely future demographic developments and the assumptions on the evolution of capital and productivity. Some hybrid versions of this approach are widely used among the policy institutions (OECD, EC, IMF). Cotis et al. (2005) propose to use the production function as

a 'flagship' method, likely to cover all policy areas, nonetheless they argue that its 'technical' performance should not be dramatically weaker compared to alternative methodologies.

DSGE models are a relatively new approach for estimating the position of business cycle. In DSGE models the concept of potential output is slightly different from the more traditional measures considered above. The backbone of the DSGE (see Gali 2015, Woodford 2003 for more) models is the Real business cycle theory, on which several imperfection (staggered price setting, monopolistic competition) are introduced. The most common measure of potential in DSGE models is the so called "flexible price" output or "natural" output which is the level of the output that would prevail if the price and wages have been fully flexible. So the related gap measures the relevance of only nominal rigidities. The other common notion is the "efficient" output which is the level of output that would prevail both in the absence of nominal rigidities and monopolistic competition. So the related gap measures the relevance of both imperfect competition and nominal rigidities. Both the natural and efficient output can be subject to different shocks (permanent and transitory productivity shocks, risk premium shocks and other shocks, mark-up shocks appear only in natural output), which makes them very volatile when applied to the data (see Vetlov et al. 2010).

3. Criteria for assessing different methods

Since output gap and potential output are not observable it is difficult to evaluate the model estimates, nevertheless to justify the use of an approach for estimation it should satisfy certain criteria. For example Jean Cotis and other economists from OECD economic department (see Cotis et al. 2005) propose the following "core requirements" i.e. the universal properties that a certain method need to have and a "user specific" requirements that are only relevant when the estimates of potential output and output gap are used for specific purposes.

1. Core requirements
    - Consistency between economic priors and the underlying assumptions of the method.
    - Transparent and easily replicated.
    - Data updates do not imply very large and unwarranted revisions in estimates.
    - The ability of a method to provide information on the precision of the estimates.
2. User specific requirements
    - Quantity and nature of information that is needed.

- End point problem.

Gran Hjelm and Kristian Jnsson from NIER (Hjelm et al (2010)), the institution which is engaged in publishing forecasts and giving fiscal and monetary policy advice on a quarterly basis, list the following quantitative and qualitative criteria for searching a best method

1. Quantitative Criteria
   - Inflation forecast performance.
   - Growth forecast performance.
   - Size of revision between the use of Quasi-Real Time (QRT) and Full Sample (FS) data.
   - Sign and change of the gaps using QRT and FS gaps.
2. Qualitative criteria
   - No end-point problem .
   - Transparent, replicable, easy to communicate internally and externally.
   - On average zero gap.

Some notable central banks also have different priorities in evaluating the estimates of potential output. For example Bank of Canada (Pichette et al. 2015) introduced the extended multivariate filter which filters separately the components of the output identified on a basis of Cob-Douglas production function and the integrated framework to project the potential growth. The banks approach focuses on the following four criteria

- An approach should provide economic interpretation.
- Estimates should be helpful in identifying inflationary and disinflationary pressures.
- The assumptions and statistical relationships conditioning potential output estimates should be consistent with the data.
- The tools should be helpful for both estimating and projecting potential.

Being one of the most transparent Central Banks the first criterion is straightforward: they need a method that would allow them to communicate the estimates easily. Though explicitly they don't mention the importance of end-sample problem, nonetheless the HP filter employed is augmented with end sample restriction not allowing the trend to change much from previous period and limiting the growth of potential from some assumed steady-state value. In contrast to above transparency and communicability considerations Bank of England's (Melolinna et al. (2016)) approach does not put a strict theoretical restriction on the concept of the output gap, since they build the measures on both macroeconomic and

financial variables, with a focus on plausibility, real-time performance and forecasting ability. The paper (Melolinna et al. (2016)) considers different unobserved components models, some with basic structure (Okun law, Phillips curve) and others augmented with different summary indicators of financial variables. The models are judged by their real time performance and inflation forecasting ability and those augmented with financial variables have better performance. Newly released staff memo (Hagelund et al. 2018) documents a set of models (7 unobserved components models and two structural VAR models) used by Norges Bank in estimating the output gap. Model estimates of output gap are evaluated by real time performance and by their ability to forecast inflation, GDP and unemployment. Average of model estimates appear to have better forecasting power.

As it becomes evident from the above mentioned literature review the chosen method of the organization is affected by the priorities considered. As a Central Bank which operates under the inflation targeting mandate we list the following criteria for evaluation of the potential output estimates. Following Cotis et al we also separate core or universal requirements that all the models should have and user specific requirements that can help to deal with certain circumstances. Note that the core requirements are mandatory only for real time policy making process. For instance if one need estimates for historic analysis there is no need for good real time performance.

1. Core criteria
    - Identification of inflationary and disinflationary pressures.
    - Small revisions between real time and final estimates.
    - Precision of the estimates.
    - Consistency with the data.
2. User specific criteria
    - Useful for projecting potential
    - Simple, economically interpretable, transparent, replicable, easy to communicate internally and externally.

First criterion is obvious. Under the inflation targeting mandate it is important to have a measure of the output gap that can provide information about the inflationary and disinflationary pressures in the economy. Second as the policy maker are more interested in the output gap at the end of sample it is important that the estimates do not change much when new data comes out. Orphanides and Van Norden (2002) document the unreliability of output gap estimates in real time using a wide range of well-known methods and show that the ex post revisions of the output gap are of the same order of magnitude as the output gap

itself thus rendering them difficult to use in policy making process. This points motivate the importance of accurately assessing the output gap and potential in real time. Next the method employed should provide precision about the estimates so that policy makers can put corresponding weight on the estimated output gap in decision making process. And finally the statistical assumptions about the unobserved components and the economic theory used should be plausible and the estimates should be consistent with the data. Among the user specific requirements we consider the ability of the model to project potential growth. As stated in Orphanides (2003) 'Real-time misperceptions about the long-run growth of the economy can play a large role in monetary policy mistakes'. Next for modern independent central banks the transparency and accountability principles have emerged as vital needs, therefore in this context the methods that will have economic interpretation and the estimates can be easily replicated and communicated would be preferable.

We do not restrict models to have on average zero gap as we do not think its an important criteria for our estimates of output gap. Haykaz Igityan (2018) documents the asymmetries in the business cycles in Armenia, which indicates that if on average inflation is on the target it is not necessary the case that output gap will be zero for the same time period.

It should be noted that there is no ideal method that can meet perfectly to this criteria and have an absolute advantage. The methods discussed in the paper should be seen as supplements to each other and can be used for different purposes. In general methodological abundance should be seen as an advantage that can provide greater scope for cross-checking diagnostics. If for example different methods provide similar results the policy makers are more confident with their estimates and if different methods provide contradicting results it creates room for discussion and selection of best tool. Besides different tools can be proved to be more effective for meeting separate criteria discussed above. For instance the multivariate filters can have good end-of-sample properties, but are poor techniques for projecting potential output compared to the production function approach.

4. Multivariate Filter

The Multivariate filter used is similar to the model proposed by Alichi and other (2017) with minor difference. It is very simple as there are only 4 blocks in the model and only data of 4 variables is required for estimation.

in the model the key macroeconomic variables(GDP (Y_t), unemployment (U_t), capacity utilization ($CAPU_t$)) are decomposed into a cyclical (defined by small letters) and a trend (marked by a bar) components.

$$y_t = Y_t - \bar{Y}_t \tag{1}$$

$$u_t = U_t - \bar{U}_t \tag{2}$$

$$capu_t = CAPU_t - C\bar{A}PU_t \tag{3}$$

In line with widely used practice potential output is assumed to follow a random walk process with time varying drift.

$$\bar{Y}_t = \bar{Y}_{t-1} + G_t + e_t^{\bar{Y}} \tag{4}$$

$$G_t = (1-\Theta)G_{t-1} + \Theta G_{ss} + e_t^G \tag{5}$$

$$y_t = \phi y_{t-1} + \phi_2 capu_t + e_t^y \tag{6}$$

Potential output evolves according to the growth rate G_t and level shock $e_t^{\bar{Y}}$. The growth rate G_t itself has some persistence G_{t-1} and it converges to a specified steady state value G_{ss} and also subject to shocks e_t^G. And finally output gap y_t has some persistence (y_{t-1}) , related with capacity utilization gap $capu_t$ contemporaneously and can be affected by demand shocks e_t^y.

The equation for inflation can be interpreted as a hybrid Phillips curve where current inflation depends on the past inflation, inflation expectations and output gap

$$\pi_t = \lambda\pi_{t-1} + (1-\lambda)\pi_{t+1} + \beta y_t + e_t^\pi - \eta e_t^{\bar{Y}} \tag{7}$$

The unemployment block describes the evolution of NAIRU $(\bar{U}_t)$ which is time varying and is allowed to deviate from steady state value U_{ss}. Growth rate of NAIRU (UG_t) is AR(1) process which has zero mean and finally Okun law relationship describes the evolution of unemployment gap (u_t) which is a function of its own lag and output gap (y_{t-1}). In contrast to the model proposed by Alichi and others the lag of output gap is used in equation (10) to describe the Okun‘s law, as our model is quarterly and it is expected that firms need some time to adjust their labor force in reaction to demand fluctuations.

$$\bar{U}_t = (1-\tau)\bar{U}_{t-1} + \tau U_{ss} + UG_t + e_t^{\bar{U}} \tag{8}$$

$$UG_t = (1-\Theta_1)UG_{t-1} + e_t^{UG} \tag{9}$$

$$u_t = \tau_2 u_{t-1} - \phi_1 y_{t-1} + e_t^u \tag{10}$$

Finally a measure of capacity utilization is incorporated to help identify information regarding the cyclical position of the economy. As suggested by many authors (Dale Jorgenson

and Zvi Griliches, Craig Burnside, Martin Eichenbaum, Sergio Rebelo) either material or energy can be used as a proxy for capital utilization. We assume that the cyclical variation in the labor utilization and capital utilization are mutually correlated and use the measure of firms electricity consumption in the economy as a proxy for capacity utilization. The steady state growth rate of trend capacity utilization (CUG_{ss}) is assumed to capture the energy efficiency trend and the coverage of the firms using electricity and it is calibrated by the mean growth rate of the electricity usage. We don't have much information about the trend capacity utilization, however it is assumed that the trend is very smooth process while the large deviations from one quarter to another capture change in utilization rate and have valuable information about the overall slack in the economy. The inclusion of the electricity consumption in the model does not change the interpretation of the potential output like in case of financial variables, so the traditional definition of the potential is still applicable to the estimates. We believe that short term movements in electricity consumption have valuable information about the cyclical position of the economy. First because Armenia is a small country with relatively large shadow economy and tax administration policies usually are performed to reduce the size of shadow economy. So in times of large tax administration policies one can see an increase in real GDP but not an increase in the electricity consumption. Second the measure of real GDP is based on complex calculations and surveys where the measurement error can be large compared to the data of electricity consumption, so in this context it can improve the precision of the estimates. And finally real time revision of GDP can be very large but the data of electricity consumption is not revised so it can help to alleviate the data revision problem. The equations describing the dynamics of capacity utilization are the following.

$$\bar{CAPU}_t = \bar{CAPU}_{t-1} + CUG_t + e_t^{\bar{CAPU}} \tag{11}$$

$$CUG_t = (1-\psi)G_{t-1} + \psi CUG_{ss} + e_t^{CUG} \tag{12}$$

$$capu_t = \kappa capu_{t-1} + e_t^{capu} \tag{13}$$

Where $CAPU_t$ is the trend consumption of electricity by firms and $capu_t$ is the gap of electricity consumption, which is assumed to capture the cyclical variation in capacity utilization.

5. Production Function Approach

Production function approach intends to provide a comprehensive economic framework for estimating the potential output and output gap. The method explicitly models the

supply side of the economy and allows to detect the sources of long-run economic growth based on the factor inputs of production and their likely future development. In practice the estimates of potential output from production function approach are considered more transparent and easy to communicate, which seems to be a strong argument for having this method as a tool for policy making process. While the estimates based on this method have economic interpretation, the question of specification of the production function is still open. The production function approach employed in this paper is a simpler version of the method used by European Commission (Havik and other(2014)).

The final output(Y_t) is assumed to be produced by a Cobb-Douglas technology by combining Labor(L_t) and Capital stock (K_t), where the latter are subject to time varying utilization rates (U_t^L and U_t^K) and efficiency levels (E_t^L and E_t^K)

$$Y_t = (U_t^L L_t E_t^L)^\alpha (U_t^K K_t E_t^K)^{1-\alpha} = L^\alpha K^{1-\alpha} TFP_t \tag{14}$$

where TFP is equal to

$$TFP_t = (U_t^L)^\alpha (U_t^K)^{1-\alpha} (E_t^L)^\alpha (E_t^K)^{1-\alpha} \tag{15}$$

which summarizes both the technological level of the factor inputs (E_t^L and E_t^K) and their utilization rate (U_t^L and U_t^K). So the conventional measure of total factor productivity which is calculated as a Solow residual from the equation (14) can be interpreted as having a cyclical part, which is the utilization of factor inputs and a trend part which is a more "precise" measure of productivity and depends on the long run trends of education, technology etc.

As a measure of Labor input the data of employment is used so the variation in the hours worked will also appear in the utilization of labor. Capital stock is estimated using the perpetual inventory method. Under the assumption of perfect competition and constant returns to scale the labor elasticity (α) is equal to the wage share in the national income and it is allowed to vary over time.

Moving from actual to potential output one need to define what is the potential level of the factor inputs and normal level of utilization. (Appendix B presents the step by step estimation of the production function approach.)

For capital it is assumed that there is no capital stock gap in the economy meaning that the actual stock of the capital represents the maximum amount of capital that can be used for production

Labor input is defined in terms of employment. For construction of trend labor ($\bar{L}_t$) input 3 components are taken into account. Trend of working age population ($\bar{WP}_t$) , trend participation rate ($\bar{PR}_t$) and trend unemployment rate ($\bar{U}$) or NAIRU. Then the potential

labor input is equal to

$$\bar{L}_t = \bar{WP}_t \bar{PR}_t (1 - \bar{U}_t) \tag{16}$$

Usually the actual working age population is taken into account while calculating the potential labor input however the data of actual working age population in Armenia is very volatile because of the seasonal migration and it can create large and unwarranted shifts in the potential output. Therefore the smoothed version of it is used for calculating the potential labor input although there is a risk of removing some valuable information while smoothing the actual working age population. Univariate Kalman filters are used for estimating the trends of the above mentioned tree components.

Potential output then can be calculated by a combination of trend inputs, their full utilization and trend efficiency. As the trend efficiency cannot be measured from data a bivariate Kalman filter is used to condition information about the cycle of TFP and the capacity utilization measure to extract the trend of the TFP ($T\bar{F}P_t$). The equation for potential is the following.

$$\bar{Y}_t = (\bar{L}_t E_t^L)^\alpha (K_t E_t^K)^{1-\alpha} = (\bar{WP}_t \bar{PR}_t (1 - \bar{U}_t))^\alpha (K_t)^{1-\alpha} T\bar{F}P_t \tag{17}$$

and the output gap is calculated as the log difference between the actual and potential output.

$$y_t = Y_t - \bar{Y}_t = \alpha(l_t) + tfp_t = \alpha(wp_t + pr_t + ln(1 - U_t) - ln(1 - \bar{U}_t)) + tfp_t \tag{18}$$

Note that output gap consists only of TFP gap and weighted labor input gap as the capital input doesn't have cyclical part.

Finally a Phillips curve specification like in Alich et al. (2017) is used to link the output gap to inflation, so that the measure of output gap is consistent to short run concept defined in section two.

$$\pi_t = \lambda \pi_{t-1} + (1 - \lambda)\pi_{t+1} + \beta y_t + e_t^\pi - \eta e_t^{T\bar{F}P} \tag{19}$$

6. Data and estimation

All the data are quarterly. Logarithmic transformation (100*log) is used for GDP, core CPI level, electricity consumption, TFP level, working age population and capital. Actual levels are used for unemployment rate, participation rate and Labor share. All the data are seasonally adjusted.

- GDP:Index of real gross domestic product at constant prices constructed from real growth rates (Statistical Committee of RA www.armstat.am)
- Core CPI: Core component of the consumer price index (Central Bank of Armenia www.cba.am)
- Electricity consumption by firms: Internal consumption of the electricity besides households and budget offices (Public services regulatory commission www.psrc.am)
- Working age population: Working age population of 15-75 years old (Statistical Committee of RA)
- Participation rate: Labor force participation rate of 15-75 years old (Statistical Committee of RA)
- Unemployment rate: Unemployment rate of 15-75 years old (Statistical Committee of RA)
- Capital stock:Calculated by author (see Appendix B) using gross capital formation data from Statistical Committee of RA.
- Labor share (total compensation of employees as a share of nominal GDP): Calculated by author (see Appendix B) using average nominal wage, employment and nominal GDP data from Statistical Committee of RA.

Bayesian estimation is used for all the models. Prior and posterior estimates of the parameters and standard deviations of the shocks are presented in the appendix of each model block. For graphical representation of the data see Appendix C.

7. Results

The aim of this section is to provide the estimations of the output gap and potential output from different models and to discuss the relative performance of the models based on the criteria discussed in section 3. By construction production function approach provides benefits in terms of economic interpretation of the estimated potential output as it gives additional decomposition of the output to factor inputs as well as technology. So the method can show which part of the potential growth is due to the underlying factors and it also becomes a natural tool for making projections or at least scenarios based on the likely future developments of the factor inputs and technology. This features are important for policy makers which makes it a 'must to have' method at policy making institutions however as argued by economists from OECD (Cotis et. al. 2005) the 'technical' performance of this methods should not be dramatically weaker across variety of criteria and the estimates from production function approach should be equally reliable in order to make it a 'preferred' method.

In this section we explore the 'technical' performance of the methods to understand the reliability and statistical accuracy of the estimates of output gap and potential output. Besides the two models documented in the previous sections we also consider the widely used methods, such as HP filter and basic multivariate filter with 3 variables (GDP, unemployment and inflation) as a benchmark for comparison.

1) HP filter
2) '3 gap' model with GDP, inflation and Unemployment
3) '4 gap' model with GDP, inflation, unemployment and capacity utilization rate
4) "PF" Production function approach

For notational simplicity we will refer to the basic multivariate filter as "3 gap" model and the multivariate filter discussed in section four as "4 gap" model.

Figure 1 shows the actual and potential output from the 4 models considered. 3 gap model is more optimistic on the level of potential output before the financial crisis and then shows a significant drop after that period compared to the 4 gap and PF approach. This is due to the embedded structure of the economic models in this methods and the contribution of the capacity utilization data in latter two methods, which indicates that resource utilization before the financial crisis was high and then there was a decline in factor utilization after the crisis. Overall different methods mostly generate the same view about the position of the business cycle in different points of time although the magnitude of the estimated output gap measures can differ significantly from one method to other.

Fig. 1. Actual and potential output

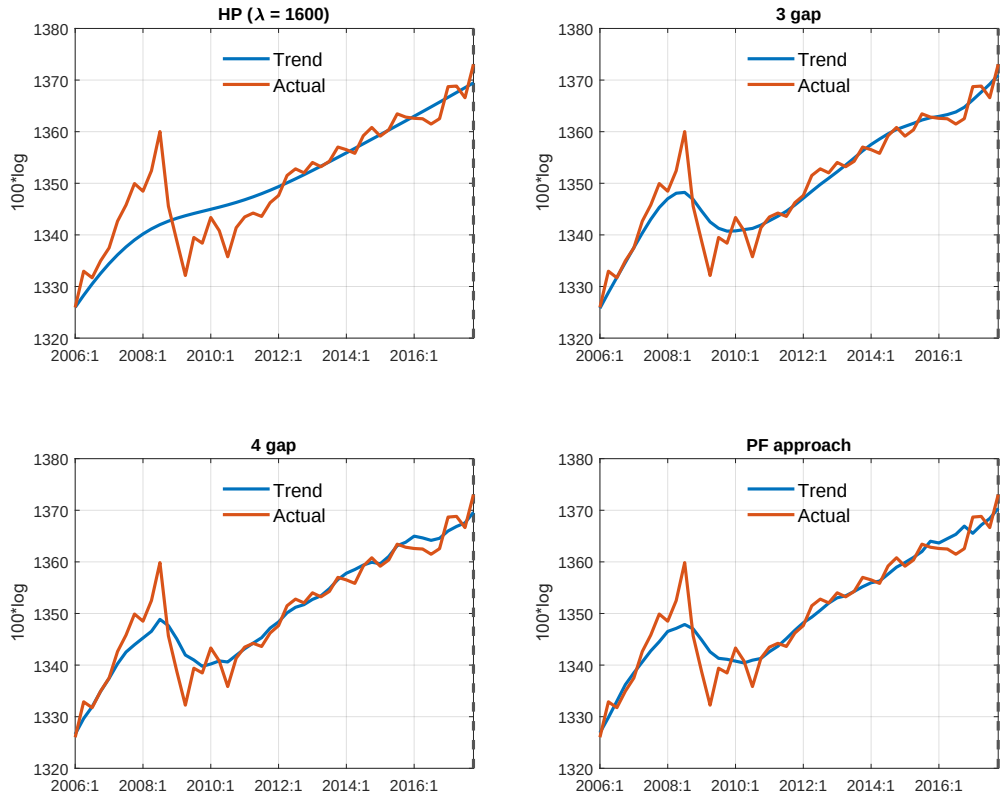
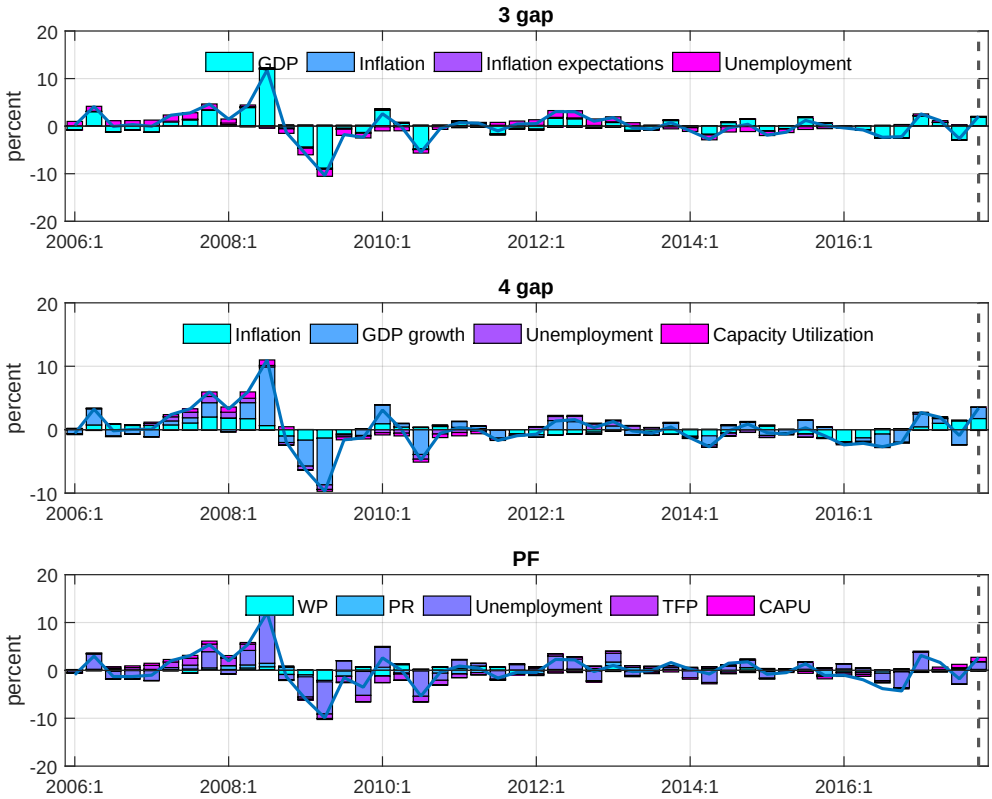


Figure 2 presents the decomposition of the smoothed output gap into different observable variables from the model. As it is common in this kind of models the main driver of the output gap in all models is the GDP itself (in case of PF approach it is TFP), however there is also significant contribution from the other variables in 4 gap and PF models, whereas in 3 gap model other observable variables have very negligible contribution. One can note the contribution of the inflation and capacity utilization to the output gap before the crisis, where ex-post measures show that there was a big positive output gap starting from the end of 2007, while in the 3 gap model the output gap was very small before 2008Q3. In general more contribution from the other variables mean that the imposed structure and variables are of greater importance which will help also to deal with revisions due to data updates. The smoothed measures from PF approach show very similar output gap estimates with respect to 4 gap model however the value added of the PF approach is the differences in the observable variables, so that one can see the decomposition of output gap from different

sides and make use of this information for cross-checking diagnostics.

Fig. 2. Contribution of the observable variables to the estimated output gap



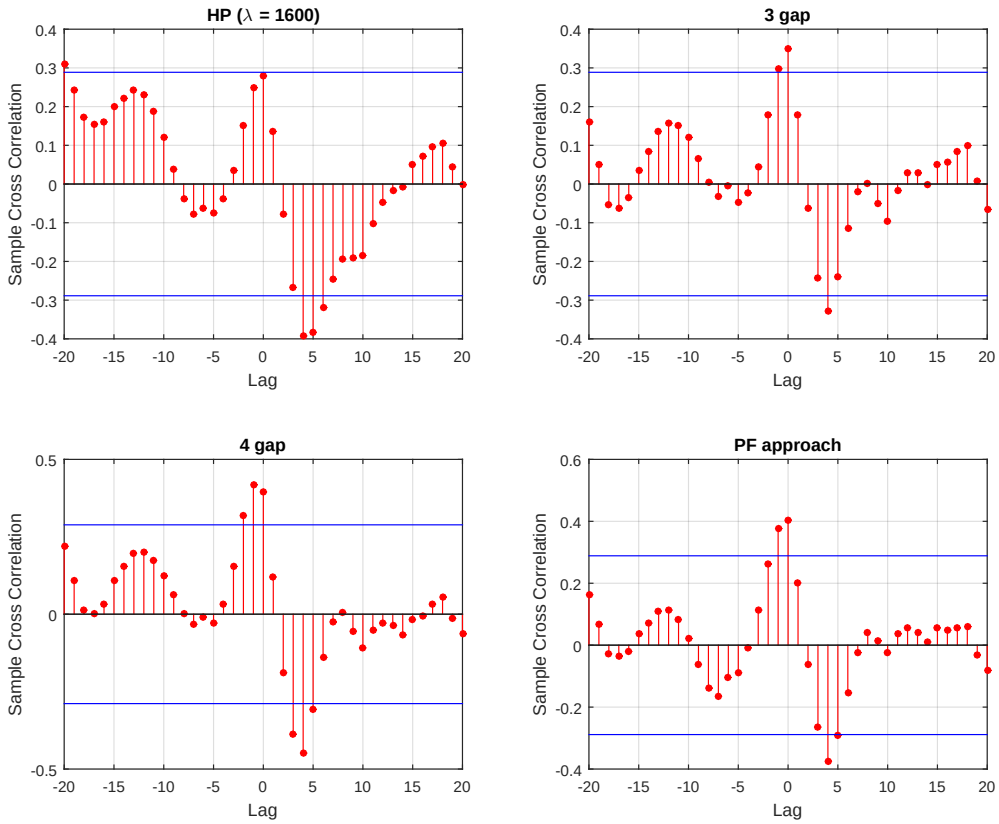
'WP is the working age population, PR is the participation rate, TFP is total factor productivity and CAPU is the capacity utilization indicator

Identification of inflationary pressures

Under the inflation targeting mandate it is of greater importance to have an estimate of output gap that contain information about the inflationary and disinflationary pressures in the economy. Figure 3 presents the cross-correlation of the inflation and the measures of output gap from the 4 methods. Several features are readily apparent. First the estimate from HP filter is not correlated with inflation at all. All other methods have significant contemporaneous correlation with inflation, nevertheless correlation is higher for the estimate from 4 gap model. Moreover the output gaps has also forecasting power in terms of inflation as they are correlated with 1 quarters ahead inflation. The negative correlation of the

inflation with a few quarters ahead output gap indicates the response from policy and the short term trade-off between inflation and output.

Fig. 3. Cross-correlation of the output gap and inflation



Blue lines indicate the confidence bands

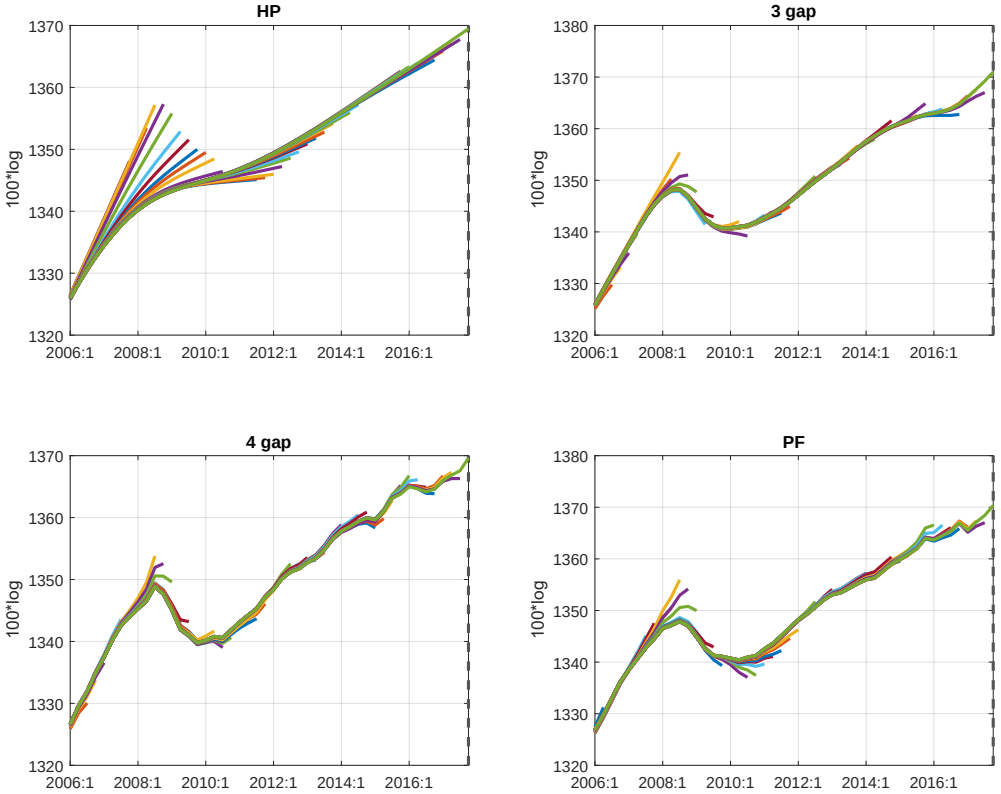
Real time performance

Given the focus of the policy makers on the the cyclical position of the economy at the end of sample period we show both ex post final and real time estimates of the output gaps and potential output. To find the source of revisions we decompose the estimates in 4 parts and using the notations of Orphanides (2002) we define the following. **Final** estimates are the smoothed estimates from the kalman estimator, meaning that the estimates of unobserved components at every point of time are conditional on the whole sample data. This simply takes the latest available vintage of data (from 2000Q1 to 2017Q4 in our case) we have and detrends it. **Quasi Final** estimates are the filtered estimates from the kalman estimator meaning that at every point of time the estimates of unobserved components are conditional

on the information set up to that point. So quasi final estimates use slightly less information (only subsamples of the final vintage data) than the final estimates, which use the full sample data. the difference between final and quasi final estimates reflect the only sample size and the magnitude of end-sample problem typical for two sided filters. Basically it shows how much the estimates of the unobserved potential output (output gap) will change when new information arrived without changing the parameters of the model and without revising the data. For construction of **Quasi Real** estimates the following steps are done. We take the subsamples of the final data and re-estimate the model for every subsample used. Initially we take the sample from 2000Q1 to 2006Q2 and estimate the models to get the output gap and potential output. then we add one more quarter and repeat the exercise up to the 2017Q4.Then we collect the last estimates from each subsample of estimation and construct the quasi real estimates by combining this last estimates. the difference between the quasi real and quasi final estimates now reflect the difference in the parameters of the models as they use the same subsample of the data but quasi final estimates are calculated based on final parameter values while the quasi real estimates are constructed by re-estimating model every time. And finally **Real Time** estimates are constructed by the same step as quasi real estimates but the real time data of GDP is used instead of sabsample from the final data. So the diference between real time estimates and quasi real time estimates reflect solely the revision of the data used for estimation (only real time data of GDP is used, for the other variables, the subsamples from final data are used).

Figure 4 presents the vintages of Quasi Final potential output estimates from the 4 models considered. this simple exercise reveals the magnitude of end-of-sample problem typical to all two-sided filters. As it is well documented in the literature the HP filter has bad performance at the end of sample and the estimates are subject to large revisions when the new information becomes available. Both 3 gap and PF methods have significant revisions, though they perform better than HP filter. Nonetheless the 4 gap model offers improvement in detecting business cycle position in real time as more identifying information is used compared to the 3 gap model. The slightly worse performance of the PF approach is not surprising as all the individual components of production function besides TFP are detrended by univariate filter. The technical performance of PF approach can be further improved by using more identifying information for filtering the separate components. This feature is important for policy makers as the measures of output gaps should provide useful guide for estimating excess demand not only in the past but more importantly at the current state of the economy. If not then this measures are meaningless and the policy makers should be aware of this drawbacks in formulating their policy choice based on the methods employed.

Fig. 4. Vintages of potential output estimates using different subsamples



subsamples of final data is used and no quarterly re-estimation of the models is performed

Table 1 presents summary revision statistics between final and quasi final estimates, where quasi final estimates are the filtered estimates from the Kalman procedure (one-sided estimates for HP filter) and they are the same as combining the last points from every subsample of estimates in figure 4. Column 1 shows the mean revision from the 4 models. On average all the methods overestimate the potential in real time. STD is the standard deviation of the revision. 4 gap model is subject to revisions less than the other methods, which is already readily apparent from figure 6. NS indicates the ratio of the standard deviation of the revision and the standard deviation of the final estimate of the potential output. this ratio gives us an indication of how severe is the end-sample problem for different models. HP filter has the worst ratio among the 4 models. CHSIGN indicates the frequency with which the quarterly change in real-time and final estimates have opposite signs. So it shows that on average 20 percent of time we think that the growth rate of potential output is positive (negative) in real time but the final estimates indicate the opposite. By this measure

4 gap model has the best performance however the differences among the models are small.

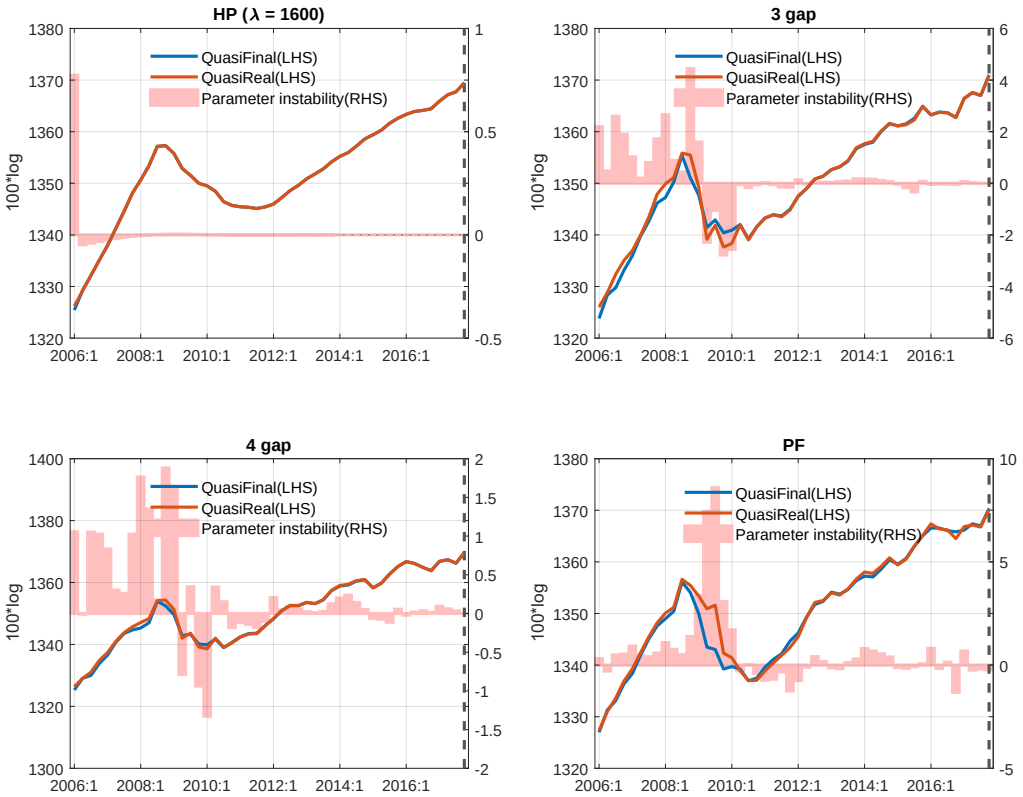
Table 1: Revision statistics: Final/quasi final

	MEAN	STD	NS	CHSIGN
HP	-1.97	4.88	0.44	0.23
3 gap	-0.23	1.60	0.14	0.17
4 gap	-0.39	1.52	0.14	0.13
PF	-0.57	2.37	0.23	0.23

Notes: This table summarize the revision statistics between final and quasi final estimates for 4 models. MEAN is the mean of the revision. STD is the standard deviation of revision. NS indicates the ratio of the standard deviation of the revision and the standard deviation of the final estimate of the potential. CHSIGN indicates the frequency with which the quarterly change in final and quasi final estimates have opposite signs.

Figure 5 shows the quasi final and quasi real estimates. In general the difference between this two estimates which is solely due to differences in the parameters of the models is very small in the recent period. Intuitively as the sample of estimation increases the Bayesian estimation employed performs better in evaluating the likelihood function and finding the posterior means of the parameters, hence they change very little as the data becomes large, while at the beginning when only 7 to 8 years of data is used for estimation more weight is being put on the prior distributions of the parameters which have relatively big standard deviations and are allowed to vary freely.

Fig. 5. Quasi final and quasi real potential output



Both methods use the same subsample of final data however the quasi real estimates perform quarterly re-estimation while the quasi final estimates uses the final parameter values for filtering potential output, so the difference between them reflects solely the stability of the model parameters

Table 2 presents comparative statistics for quasi final and quasi real estimates for 4 models. 4 gap model has more equations and hence more parameters to estimates, but nevertheless the comparison of STD and NS columns from the table reveals that its parameters are more stable than the ones from 3 gap model. CHSIGN column shows that the change in parameters of the model has negligible effect for the sign in potential growth rates meaning that the quality of information is robust to changes in parameters. For HP filter the λ parameter is calibrated to 1600, so it is not comparable in this context.

Figure 6 shows the quasi real and real time estimates from the 4 models and reveals the importance of data revision for conduction of monetary policy in real time. As we see the data revision is almost equally important for 4 models however the problem is more severe for HP filter, which uses only data of GDP for estimation, while the other methods use more

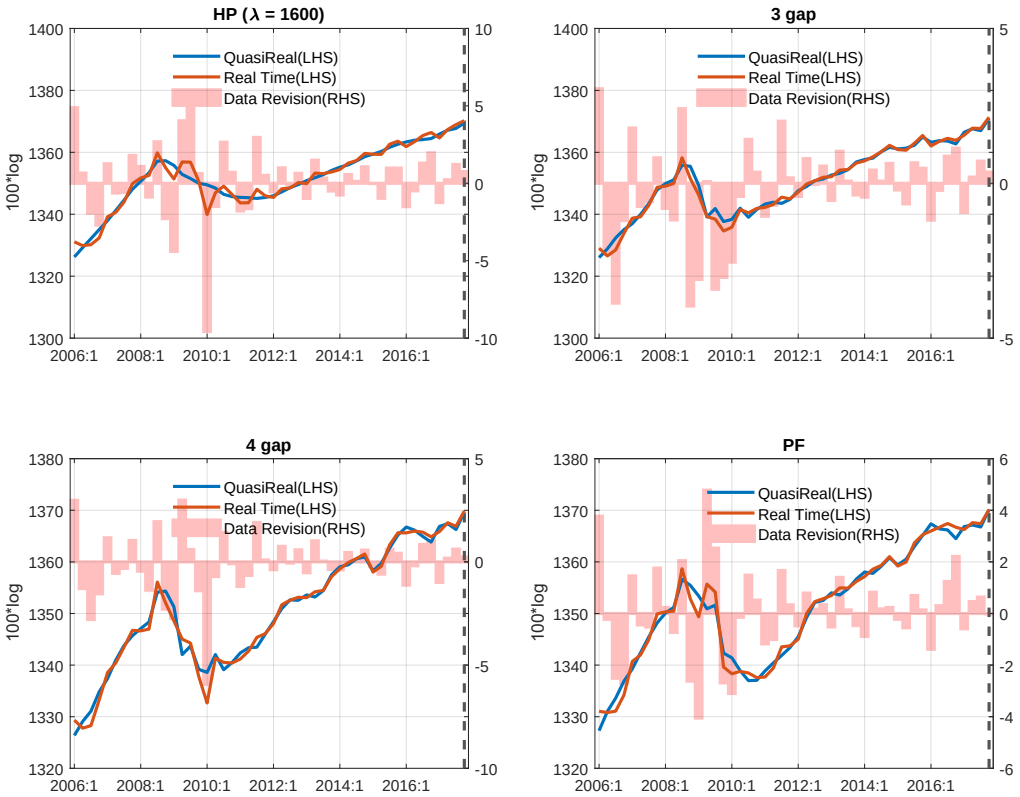
Table 2: Revision statistics: Quasi final/quasi real

	MEAN	STD	NS	CHSIGN
HP	-0.01	0.11	0.01	0.00
3 gap	-0.25	1.21	0.11	0.00
4 gap	-0.22	0.62	0.06	0.02
PF	-0.61	1.79	0.17	0.04

Notes: This table summarize the revision statistics between quasi final and quasi real estimates for 4 models. the relevant sample is from 2006Q2 to 2017Q4. MEAN is the mean of the revision. STD is the standard deviation of the revision. NS indicates the ratio of the standard deviation of the revision and the standard deviation of the final estimate of the potential. CHSIGN indicates the frequency with which the quarterly change in Quasi final and quasi real estimates have opposite signs.

identifying information and are less vulnerable for data revisions.

Fig. 6. Quasi real and real time potential output



Both methods re-estimate the models using subsamples of the data, but the real time estimates are based on the real time GDP data while quasi real estimates are based on a subsamples of final data. So the difference between them reflects the data revision.

Table 3 shows the summary statistics for quasi real and real time estimates. As it becomes apparent from the table mean revision is slightly positive as the data revision itself. Size of the revision (STD) is of the same magnitude as the data revision for HP filter. Other methods are less vulnerable to data revision due to contribution of other variables in the estimation process. NS and CHSIGN columns show that the revision of estimates due to data revisions is a bigger issue than the parameter instability and the end-sample problem.

Figure 7 and Table 4 shows the Final and real time estimates and presents the total revision from the 4 models considered, which is due to all the factors considered above separately (end-sample problem, data revision and parameter instability). In general one can note that all the models are less reliable during an upturn especially during the financial crisis. Nevertheless 4 gap and 3 gap models perform better and 4 gap shows some improvement

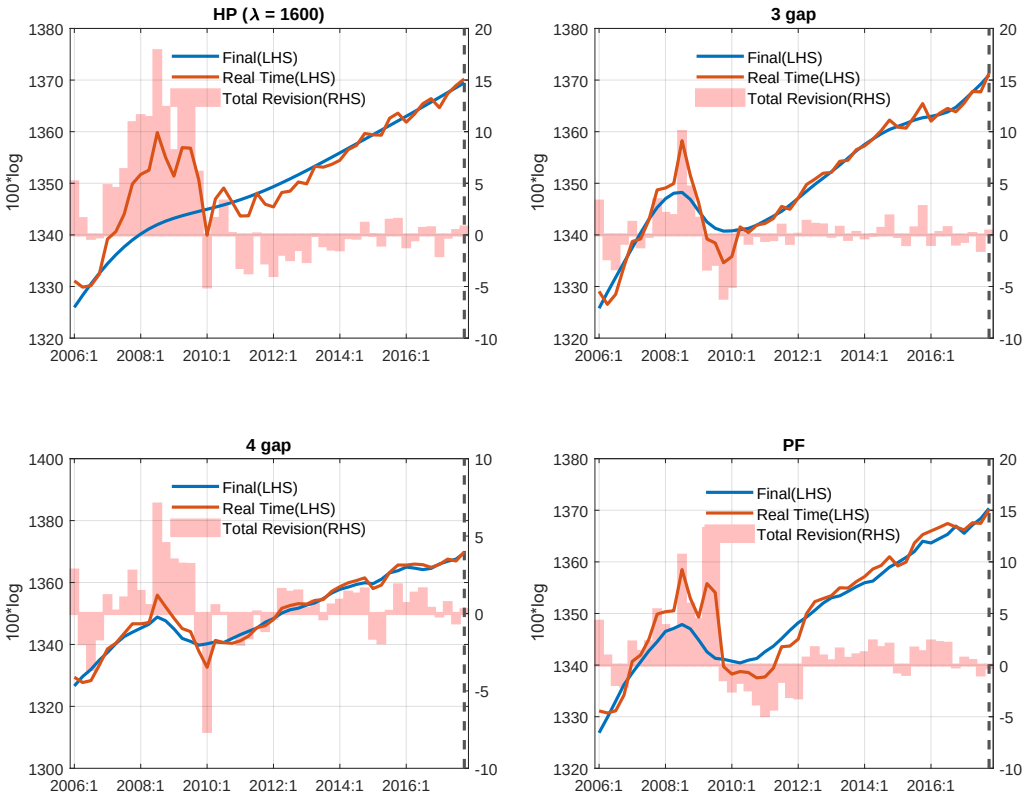
Table 3: Revision statistics: Quasi real/ real time

	MEAN	STD	NS	CHSIGN
HP	0.04	2.25	0.21	0.31
3 gap	0.36	1.47	0.39	0.13
4 gap	0.20	1.43	0.33	0.21
PF	0.01	1.59	0.35	0.23
Data Revision	0.24	2.13		

Notes: This table summarize the revision statistics between quasi real and real time estimates for 4 models. the relevant sample is from 2006Q2 to 2017Q4. MEAN is the mean of the revision. STD is the standard deviation of the revision. NS indicates the ratio of the standard deviation of the revision and the standard deviation of the final estimate of the potential. CHSIGN indicates the frequency with which the quarterly change in Quasi real and real time estimates have opposite signs.

in terms of revision size. In line with criticism of HP filter in the literature this exercise shows the unreliability of HP filter for real time policy analysis. Production approach is more reliable than HP filter but its performance is somewhat weaker compared to the 4 gap and 3 gap models. the detailed examination of the revision of estimates shows that there can be substantial gains from using relevant information for identification of the unobserved variables, especially by reducing the end-sample problem and by alleviating the data revision problem.

Fig. 7. Final and real time potential output



Final smoothed series from Kalman procedure and real time estimates with real time GDP data and quarterly re-estimation of the models. the relevant sample is from 2006Q2 to 2017Q4.

Table 4: Revision statistics: Final/Real time

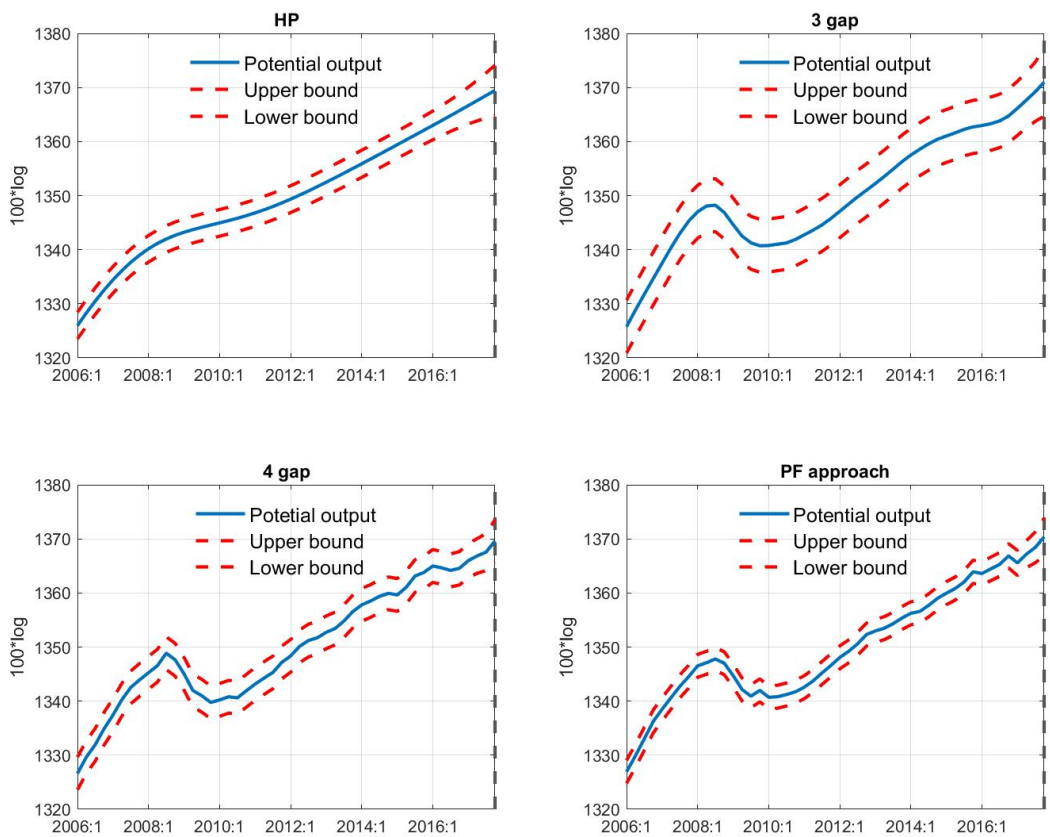
	Mean	STD	NS	CHSIGN
HP	-2.05	5.31	0.50	0.33
3 gap	-0.09	2.42	0.23	0.17
4 gap	-0.40	2.22	0.21	0.19
PF	-1.18	3.77	0.36	0.21

Precision of the estimates

As noted in the beginning the useful method should provide information about the precision of the estimates so that the policy makers can put corresponding weight on the output gap or the potential output in decision making process. the Kalman filter employed for estimation allows to calculate confidence bands around the estimates however it ignores the effects of

data revisions and model misspecification. So only if the these two are relatively very small then these measure of uncertainty can be a useful guide. Figure 8 shows the estimated confidence intervals around the smoothed potential output from the 4 model. estimates from 4 gap model has narrower confidence bands than 3 gap model and HP filter while PF approach have more confidence in the results than the 4 gap model, however this may not be correct conclusion. As stated above these measures ignore the specification of the model and assume that they are all correct.

Fig. 8. 95 % confidence bands around the smoothed potential outputs

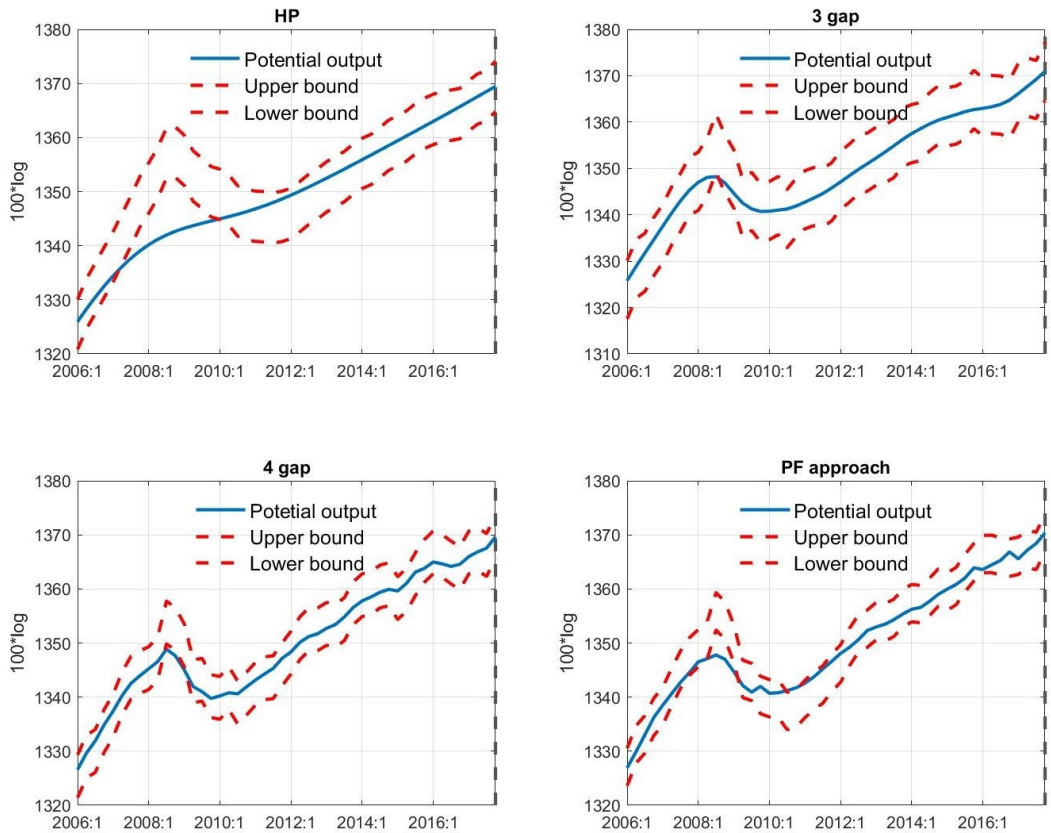


Red lines indicate the 95% confidence intervals

To argue on this point further figure 9 presents the confidence bands around the filtered potential output (as a proxy for real time estimate) and the final smoothed potential output from the 4 models. as it is apparent from the graph the smoothed potential output from the 4 gap and 3 gap models tracks the filtered estimate quite well and stays within the confidence bands almost at entire sample while HP filter don‘t capture the uncertainty in real time as

the final estimates are out of confidence bands for quite a long period. Beside a few quarters the PF approach also captures the uncertainty quite well.

Fig. 9. 95 % confidence bands around the filtered potential outputs and final estimates



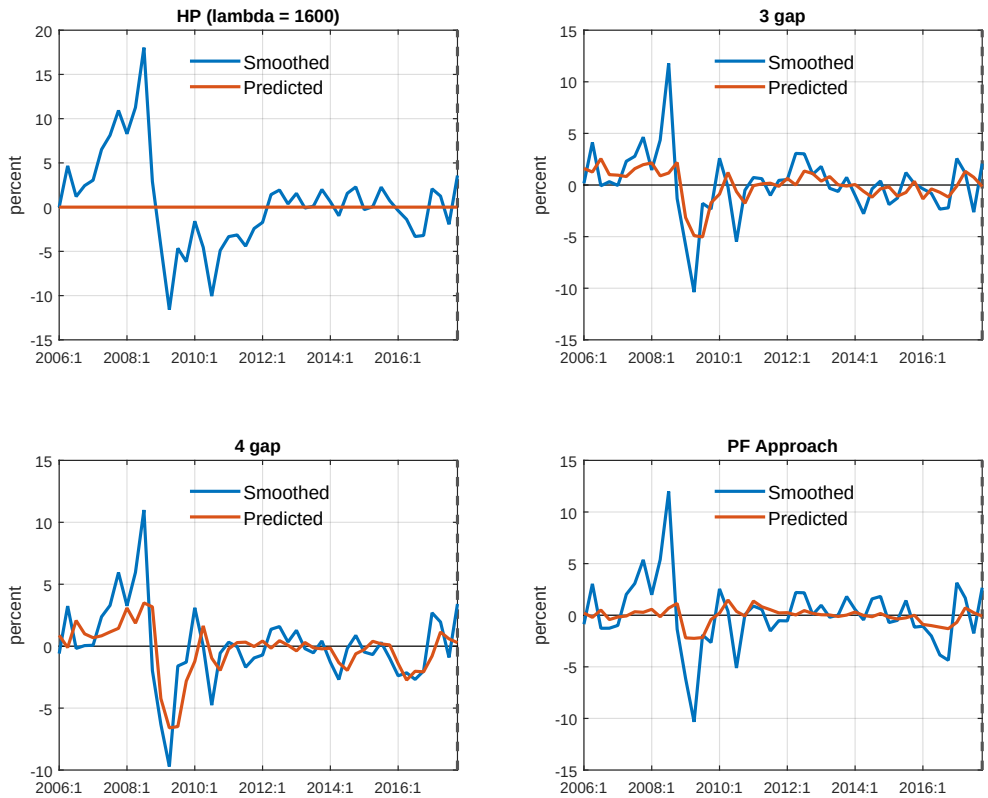
Red lines indicate the 95% confidence intervals around filtered estimates

Underlying assumptions of the model and consistency with data

It is difficult to evaluate how consistent are the estimates of unobserved variables with the data as no actual data of output gap (potential output) exist. Some authors try to compare the output gaps with the capacity utilization indicator which are measured by national statistical services, however such indicator for Armenia does not exist and a proxy for capacity utilization is already included in the estimation process. So the consistency should be based on policy makers historical beliefs about the cyclical position of the economy, nevertheless some judgment about statistical properties of the models can be made while looking at the Smoothed and predicted output gaps (Figure 10) from the 4 models. Definitely the white noise assumption underlying the HP filter is a poor approximation of the output

gap process as it obviously have some persistence. that‘s why the predictions from HP filter are always zero. It is difficult to judge about the other methods however the 4 gap model seems to have more realistic statistical assumptions and specification as the predicted values of output gap are tracking final estimates quite closely.

Fig. 10. Smoothed and predicted output gaps



’Smoothed’ series at every point are estimates based on the all information set while ’predicted’ series are one step ahead predictions based on the information set available up to that point.

User specific requirements

Regarding the user specific requirements it is even more difficult to compare the models. Of course there is no much difference between 3 gap and 4 gap models in terms of the transparency, replicability and ease of communication. The additional information (utilization rate which is proxied by electricity consumption of enterprises) used in the 4 gap model does not change the interpretation of the output gap like in case of adding financial variables in the model (Melonina et al. 2016) and as soon as all the assumptions are justified and the steps of estimation are explained the models should be judged only by core performance. HP

filter is a simpler and more transparent method, which is easy to replicate. This user specific requirements seem to be strong arguments which proves such a widespread usage of HP filter even for real time policy making process (The Riksbank published output gap solely on HP filter (Hjelm et al. (2010))). The production function approach is also estimated with Kalman procedure with conditioning the TFP on the utilization rate, so from 'bird's eye perspective' it can not be considered simpler and more transparent approach however it provides additional economic structure by separating the output into factor components thus making the estimates of potential output easy to communicate. Besides it is used by notable economic organizations as a main tool for estimating potential output (output gap), which can also increase the transparency of the method and allow the user to compare the country results with the ones estimated by those institutions. Besides it is easier to project or at least create scenarios of potential growth based on the likely future developments of the factor inputs.

8. Conclusion

The notions of potential output and output gap are crucial in many areas of economic policy and are used very extensively among the policymakers however the measurement of the latter unobserved variables is not an easy task. The difficulty arises especially in measuring them in real time.

The paper documents some methods for estimating the potential output and output gap, discusses some relevant criteria for assessing the different models and presents the comparative results for the models considered. The main findings are the follows.

The introduction of the capacity utilization rate in the basic unobserved components model offers improvements in a set of criteria and makes the method the best tool in the context of core requirements.

Real time performance of all models has been improved in recent periods and the estimates from these models are more reliable now than they were in a few years ago.

historical updates of GDP Data are still an important part of the overall revisions of the estimates. Model comparisons show that more relevant variables that are used for identifying the state of unobserved components can alleviate the dependence of the estimates on the data revision.

The production function approach can be proved to be more suitable for user specific requirements (for example projecting the potential growth based on the likely future development of the factor inputs and productivity) and is widely used among different notable institutions, which can make it more transparent and easy to communicate, however the

technical performance of this model is slightly worse than the multivariate filters but far better than the HP filter.

Appendix A. Multivariate filters

A.1. 3 gap model, GDP, Unemployment, Inflation

3 gap model presented in this section is similar to the one proposed by Benes et. al. (2010).

Definition of GDP (Y_t)

$$y_t = Y_t - \bar{Y}_t \tag{20}$$

Process for potential output

$$\bar{Y}_t = \bar{Y}_{t-1} + G_t - \alpha(\bar{U}_t - \bar{U_{t-1}}) \tag{21}$$

Process for potential growth

$$G_t = (1-\theta)G_{t-1} + \theta G_{ss} + e_t^G \tag{22}$$

process for output gap

$$y_t = \phi y_{t-1} + \phi_1/100(\pi_{t-1} - \pi_{t-1}^E) + e_t^y \tag{23}$$

Process for inflation

$$\pi_t = \lambda\pi_{t-1} + (1-\lambda)\pi_{t-1}^E + \beta y_t + \beta_1(y_t - y_{t-1}) + e_t^\pi \tag{24}$$

Definition of unemployment rate (U_t)

$$\bar{U}_t = U_t + u_t \tag{25}$$

Process for trend unemployment

$$\bar{U}_t = \bar{U}_{t-1} + UG_t - \tau_4/100(\bar{U}_{t-1} - U_{ss}) - \tau_5 y_{t-1} \tag{26}$$

Process for growth rate of trend unemployemnt

$$UG_t = (1-\tau_3)UG_{t-1} + e_t^{UG} \tag{27}$$

Process for unemployment gap

$$u_t = \tau_1 y_{t-1} + \tau_2 u_{t-1} + e_t^u \tag{28}$$

Process for inflation expectations

$$\pi_t^E = \pi_{t-1}^E + e_t^{\pi_E} \tag{29}$$

Table 5: Estimated Parameters. Estimation period: 2000Q1-2017Q4

Parameters	Mode		Support		Dispersion	
	Prior	Posterior	Low	High	Prior	Posterior
τ_3	0.10	0.11	0.05	0.99	0.05	0.04
β	0.30	0.30	0.01	0.90	0.01	0.01
β_1	0.40	0.31	0.00	0.90	0.20	0.18
λ	0.80	0.25	0.01	0.90	1.00	0.76
ϕ	0.60	0.54	0.10	0.95	0.10	0.08
τ_2	0.50	0.47	0.05	0.90	0.05	0.05
τ_1	0.25	0.10	0.10	0.99	0.05	0.03
θ	0.30	0.30	0.00	0.60	0.01	0.01
ϕ_1	5.00	5.20	2.00	0.98	1.00	1.05
τ_4	2.00	2.01	0.50	10.00	0.20	0.21
τ_5	0.20	0.07	0.00	20.00	0.20	0.04
$\sigma(e_t^G)$	0.40	0.91	0.01	0.40	0.30	0.14
$\sigma(e_t^y)$	2.00	2.78	0.01	3.00	0.50	0.22
$\sigma(e_t^u)$	1.50	0.73	0.01	3.00	0.50	0.07
$\sigma(e_t^{UG})$	0.30	0.14	0.01	3.00	0.10	0.05
$\sigma(e_t^\pi)$	10.00	9.99	0.10	15.00	0.03	0.03
$\sigma(e_t^{\pi_E})$	0.20	0.93	0.00	1.00	0.05	0.26

*Calibrated parameters: $G_{ss} = 1.125$, $U_{ss} = 17.5$

A.2. 4 gap model, GDP, Unemployment, Inflation, Utilization rate

Definition of GDP (Y_t)

$$y_t = Y_t - \bar{Y}_t \tag{30}$$

Process for potential output

$$\bar{Y}_t = \bar{Y}_{t-1} + G_t + e_t^{\bar{Y}} \tag{31}$$

Process for potential growth

$$G_t = (1-\theta)G_{t-1} + \theta G_{ss} + e_t^G \tag{32}$$

process for output gap

$$y_t = \phi y_{t-1} + \phi_2 capu_t + e_t^y \tag{33}$$

Process for inflation

$$\pi_t = \lambda\pi_{t-1} + (1-\lambda)\pi_{t+1} + \beta y_t + e_t^\pi - \eta e_t^{\bar{Y}} \tag{34}$$

Definition of unemployment rate (U_t)

$$u_t = U_t - \bar{U}_t \tag{35}$$

Process for trend unemployment

$$\bar{U}_t = (1-\tau)\bar{U}_{t-1} + \tau U_{ss} + UG_t + e_t^{\bar{U}} \tag{36}$$

Process for growth rate of trend unemployemnt

$$UG_t = (1-\Theta_1)UG_{t-1} + e_t^{UG} \tag{37}$$

Process for unemployment gap

$$u_t = \tau_2 u_{t-1} - \phi_1 y_{t-1} + e_t^u \tag{38}$$

Definition of capacity utilization rate ($CAPU_t$)

$$capu_t = CAPU_t - C\bar{A}PU_t \tag{39}$$

Process for trend capacity utilization

$$C\bar{A}PU_t = C\bar{A}PU_{t-1} + CUG_t + e_t^{C\bar{A}PU} \tag{40}$$

Process for growth rate of trend capacity utilization

$$CUG_t = (1-\psi)G_{t-1} + \psi CUG_{ss} + e_t^{CUG} \tag{41}$$

Process for capacity utilization gap

$$capu_t = \kappa capu_{t-1} + e_t^{capu} \tag{42}$$

Table 6: Estimated Parameters. Estimation period: 2000Q1-2017Q4

Parameters	Mode		Support		Dispersion	
	Prior	Posterior	Low	High	Prior	Posterior
λ	0.20	0.13	0.05	0.99	0.10	0.06
β	0.30	0.30	0.01	3.00	0.01	0.01
ϕ	0.60	0.50	0.10	0.99	0.10	0.08
ϕ_1	0.20	0.12	0.10	0.99	0.10	0.03
ϕ_2	0.20	0.22	0.10	0.99	0.10	0.07
θ	0.30	0.21	0.05	0.99	0.10	0.06
τ_2	0.50	0.46	0.05	0.99	0.05	0.05
τ_3	0.10	0.06	0.05	0.99	0.05	0.03
τ_4	0.10	0.06	0.05	0.99	0.05	0.03
κ	0.50	0.47	0.05	1.00	0.10	0.09
η	0.50	0.48	0.05	1.00	0.10	0.10
δ	0.25	0.22	0.05	0.99	0.10	0.08
$\sigma(e_t^{\bar{Y}})$	1.00	1.05	0.01	4.00	0.50	0.36
$\sigma(e_t^{G})$	0.40	0.78	0.01	3.00	0.30	0.16
$\sigma(e_t^{y})$	2.00	2.54	0.01	3.00	0.50	0.22
$\sigma(e_t^{\pi})$	1.20	1.47	0.01	3.00	0.50	0.18
$\sigma(e_t^{u})$	1.50	0.65	0.01	3.00	0.50	0.07
$\sigma(e_t^{\bar{U}})$	0.50	0.11	0.01	3.00	0.50	0.28
$\sigma(e_t^{UG})$	0.30	0.12	0.01	3.00	0.30	0.03
$\sigma(e_t^{C\bar{A}PU})$	0.50	0.50	0.01	0.99	0.50	4.00
$\sigma(e_t^{CUG})$	0.30	0.42	0.01	0.99	0.30	0.14
$\sigma(e_t^{capu})$	4.00	3.84	1.00	10.00	1.00	0.36

*Calibrated parameters: $G_{ss} = 1.125$, $U_{ss} = 17.5$, $CU_{ss} = 1.125$

A.3. HP Kalman filter

An often used specification of Hodrick-Prescott filter is penalized least squares form

$$min \sum_{t=-\infty}^{\infty} (y_t - \bar{y}_t)^2 + \sum_{t=-\infty}^{\infty} \lambda[(\bar{y_{t+1}} - \bar{y}_t) - (\bar{y}_t - \bar{y_{t-1}})]^2 \tag{43}$$

where the first term penalizes the cyclical component and the second term penalizes the variation in the growth rate of trend component. Higer λ means higher penalty for variation in the growth rate of the trend i.e. smoother trend.

HP filter can be modeled alternatively as an unobserved components model as

$$y_t = \bar{y}_t + x_t \tag{44}$$

$$x_t = e_t^x \quad e_t^x \ N(0, \sigma_x^2) \tag{45}$$

$$\bar{y_{t+1}} - \bar{y}_t = \bar{y}_t - \bar{y_{t-1}} + e_t^g \quad e_t^g \ N(0, \sigma_g^2) \tag{46}$$

Where the output gap is assumed to be white noise and trend is assumed to be an I(2) process. For large samples HP filter estimation is the same as Kalman filter estimation of this model when $\lambda = \sigma_g^2/\sigma_x^2$ (signal-noise ratio).

Appendix B. Detailed technical description of production function approach

This appendix shows the step by step technical procedure for estimating the potential output by production function approach First the data of capital is estimated using the perpetual inventory method. Capital accumulation law is used to construct the capital stock from the investment data and 2.5 percent quarterly deprecation rate is assumed. Initial stock of capital is estimated assuming a steady state ratio of capital to GDP. It is assumed that there is no capital stock gap in the economy meaning that the actual stock of the capital is the maximum amount of the capital that can be used for production.

After constructing the data of capital the growth rate of TFP (ΔTFP_t) is calculated as a residual using the growth accounting framework.

$$\Delta TFP_t = \Delta Y_t - \alpha \Delta L_t - (1 - \alpha)\Delta K_t \tag{47}$$

where α is the elasticity of GDP with respect to labor input which is time varying. Under certain assumptions one can show that the α is equal to the labor income share from the distribution of incomes and the data for α is obtained by dividing the whole-economy nominal wage (average nominal wage multiplied by number of employees) to the nominal GDP.

In this context the TFP is generally not considered to be a measure of productivity but its a residual and captures many other things beside 'pure' productivity. As the growth accounting framework allows as to calculate only the growth rate of TFP, the level index is constructed using growth rates and assuming an index number 100 at the beginning of the period. Then a bivariate Kalman filter is employed to exploit the link between TFP and capacity utilization measure and to extract the trend TFP which is a better approximation to trend productivity. The version of the model for extracting trend and cycle of TFP consists of the following equations.

$$TFP_t = T\bar{F}P_t + tfp_t \tag{48}$$

$$T\bar{F}P_t = T\bar{F}P_{t-1} + TFPG_t + e_t^{T\bar{F}P} \tag{49}$$

$$TFPG_t = (1-\rho)TFPG_{t-1} + \rho TFPG_{ss} + e_t^{TFPG} \tag{50}$$

$$tfp_t = 2\phi cos(2\pi/\theta)tfp_{t-1} - \phi^2 tfp_{t-2} + e_t^{tfp} \tag{51}$$

$$CAPU_t = C\bar{A}PU_t + capu_t \tag{52}$$

$$C\bar{A}PU_t = C\bar{A}PU_{t-1} + CUG_t + e_t^{C\bar{A}PU} \tag{53}$$

$$CUG_t = (1-\delta)G_{t-1} + \delta CUG_{ss} + e_t^{CUG} \tag{54}$$

$$capu_t = \kappa tfp_t + e_t^{capu} \tag{55}$$

Where the variables with bar represent the trend component and variables with small letter represent the cyclical component.

Labor input is defined in terms of employment. For construction of labor (L_t) input 3 components are taken into account. Working age population (WP_t) , participation rate (PR_t) and the unemployment rate (U_t). Then the labor input is equal to

$$L_t = WP_t PR_t(1-U_t) \tag{56}$$

The same relationship holds for trend labor ($\bar{L}_t$) input. Usually the actual working age population is used to construct the trend labor input but the actual data is very volatile due to seasonal migration. So in order to avoid such unwarranted shifts in potential output the trend of working age population ($W\bar{P}_t$) is used instead, however there is a risk of removing substantial information while doing this.

$$\bar{L}_t = W\bar{P}_t P\bar{R}_t(1-\bar{U}_t) \tag{57}$$

The trend and gap of each of the tree components (WP_t,PR_t,U_t) is estimated using a univariate Kalman filter approach. The following equations describe the evolution of trend and cycle of working age population (WP_t) , participation rate (PR_t) and the unemployment rate (U_t).

$$WP_t = \bar{WP}_t + wp_t \tag{58}$$

$$\bar{WP}_t = \bar{WP}_{t-1} + WPG_t + e_t^{\bar{WP}} \tag{59}$$

$$WPG_t = (1 - \rho_{WP})WPG_{t-1} + \rho_{WP}WPG_{ss} + e_t^{WPG} \tag{60}$$

$$wp_t = \phi_{WP}wp_{t-1} + e_t^{wp} \tag{61}$$

$$PR_t = \bar{PR}_t + pr_t \tag{62}$$

$$\bar{PR}_t = (1 - \rho_{PR})\bar{PR}_{t-1} + PRG_t + \rho_{PR}PRG_{ss} + e_t^{\bar{PR}} \tag{63}$$

$$PRG_t = (1 - \rho_{PRG})PRG_{t-1} + e_t^{PRG} \tag{64}$$

$$pr_t = \phi_{pr}pr_{t-1} + e_t^{pr} \tag{65}$$

$$U_t = \bar{U}_t + u_t \tag{66}$$

$$\bar{U}_t = (1 - \rho_U)\bar{U}_{t-1} + UG_t + \rho_U UG_{ss} + e_t^{\bar{U}} \tag{67}$$

$$UG_t = (1 - \rho_{UG})UG_{t-1} + e_t^{UG} \tag{68}$$

$$u_t = \phi_u u_{t-1} + e_t^u \tag{69}$$

Trend working age population,trend participation rate and trend unemployment rate can also be easily handled with HP filter, however the Kalman filter approach is employed for technical convenience as the HP filter doesn't allow to calculate the confidence intervals for the unobserved components as opposed to Kalman filter.

Then using the Cobb-Douglas technology potential output is equal to

$$\bar{Y}_t = (\bar{L}_t)^{\alpha}(K_t)^{1-\alpha}T\bar{F}P_t = (\bar{WP}_t\bar{PR}_t(1 - \bar{U}_t))^{\alpha}(K_t)^{1-\alpha}T\bar{F}P_t \tag{70}$$

and output gap is calculated as the log difference between actual and potential output

$$y_t = Y_t - \bar{Y}_t = \alpha(l_t) + tfp_t = \alpha(wp_t + pr_t + ln(1 - U_t) - ln(1 - \bar{U}_t)) + tfp_t \tag{71}$$

Note that output gap consists only of TFP gap and weighted labor input gap as the capital input doesn't have cyclical part.

Finally a Phillips curve specification is used to link the output gap to inflation

$$\pi_t = \lambda\pi_{t-1} + (1-\lambda)\pi_{t+1} + \beta y_t + e_t^{\pi} - \eta e_t^{T\bar{F}P} \tag{72}$$

Note that the TFP is an index and if one wants to calculate the potential output directly from equation (42) then one needs to calculate the growth rate of trend output first by combining the growth rates of trend TFP, trend Labor and capital inputs and use a judgment to come up with an estimate of the level of potential output. For instance one can make a judgment to identify a certain period where it is believed that the trend output is equal to actual output and then construct the level of the potential output by growth rates. The next option is just to calculate the output gap first from the equation (43) and subtract the output gap from the actual output to come up with a measure of potential output.

The system of equations above can be represented by a state space form and can be estimated with Kalman filter easily.

$$Y_t = AX_t \tag{73}$$

$$X_t = BX_{t-1} + CW_t \tag{74}$$

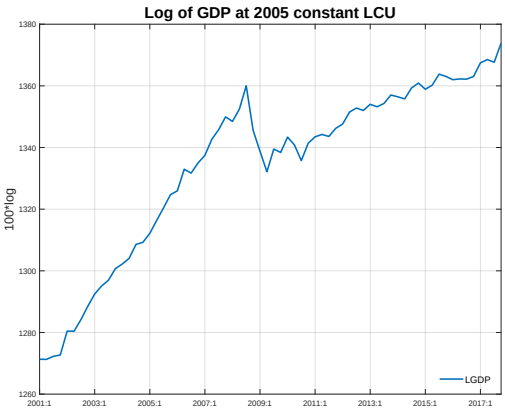
Where the first equation is the measurement equation which links the observables Y_t (WP_t, TFP_t, PR_t, U_t) to unobserved X_t variables ($T\bar{F}P_t$, tfp_t, $C\bar{A}PU_t$, $capu_t$, $\bar{WP}_t$, wp_t, $\bar{PR}_t$, pr_t, $\bar{U}_t$, u_t,) and the second equation describes the dynamics of the unobserved components, where the W_t is a matrix of random innovations, which are not mutually correlated.

Bayesian approach is employed to estimate the parameters of the model. Table 7 shows the estimated parameters.

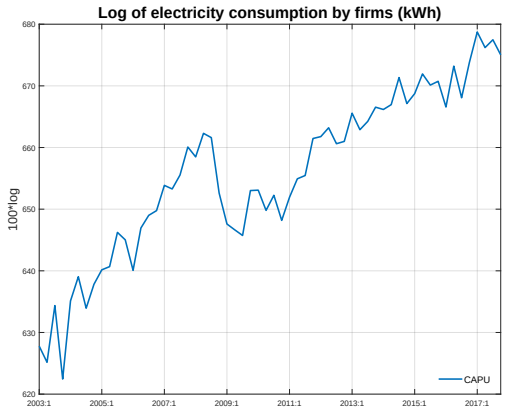
Table 7: Estimated Parameters. Estimation period: 2000Q1-2017Q4

Parameters	Mode		Support		Dispersion	
	Prior	Posterior	Low	High	Prior	Posterior
ρ	0.10	0.07	0.01	0.99	0.10	0.04
ρ_{WP}	0.10	0.08	0.01	0.99	0.10	0.06
ρ_{PR}	0.10	0.06	0.01	0.99	0.10	0.04
ρ_{PRG}	0.10	0.07	0.01	0.99	0.10	0.04
ρ_U	0.10	0.02	0.01	0.99	0.10	0.02
ρ_{UG}	0.10	0.04	0.01	0.99	0.10	0.02
θ	6.00	5.41	2.00	32.00	3.50	1.51
$TFPG_{ss}$	1.00	1.02	0.01	7.00	0.10	0.10
CUG_{ss}	1.50	1.50	0.01	7.00	0.00	0.00
ϕ	0.42	0.27	0.10	0.99	0.17	0.10
ϕ_{WP}	0.42	0.37	0.10	0.99	0.17	0.13
ϕ_{PR}	0.42	0.33	0.10	0.99	0.17	0.11
ϕ_U	0.42	0.53	0.10	0.99	0.17	0.13
κ	1.00	0.89	0.01	3.00	0.10	0.08
δ	0.10	0.04	0.01	3.00	0.10	0.03
β	0.30	0.19	0.01	3.00	0.10	0.05
λ	0.20	0.29	0.01	3.00	0.10	0.05
$\sigma(e_t^{\bar{TFP}})$	0.00	0.00	0.00	5.00	0.00	0.00
$\sigma(e_t^{TFPG})$	0.60	2.64	0.00	10.00	0.60	0.21
$\sigma(e_t^{tfp})$	0.10	0.43	0.00	0.99	0.05	0.07
$\sigma(e_t^{\bar{WP}})$	0.00	0.00	0.00	5.00	0.00	0.00
$\sigma(e_t^{wp})$	0.60	2.14	0.00	10.00	0.60	0.23
$\sigma(e_t^{WPG})$	0.10	0.06	0.00	0.99	0.10	0.02
$\sigma(e_t^{\bar{PR}})$	0.00	0.00	0.00	5.00	0.00	0.00
$\sigma(e_t^{pr})$	0.60	1.19	0.00	10.00	0.60	0.11
$\sigma(e_t^{PRG})$	0.10	0.11	0.00	0.99	0.10	0.03
$\sigma(e_t^{\bar{U}})$	0.00	0.00	0.00	5.00	0.00	0.00
$\sigma(e_t^{u})$	0.30	1.03	0.00	10.00	0.60	0.10
$\sigma(e_t^{UG})$	0.10	0.08	0.00	0.99	0.10	0.02
$\sigma(e_t^{\bar{CAPU}})$	0.00	0.00	0.00	5.00	0.00	0.00
$\sigma(e_t^{capu})$	0.60	3.25	1.00	10.00	0.60	0.33
$\sigma(e_t^{CUG})$	0.15	0.19	0.00	5.00	0.05	0.05
$\sigma(e_t^{\pi})$	0.70	1.29	0.00	5.00	0.30	0.12

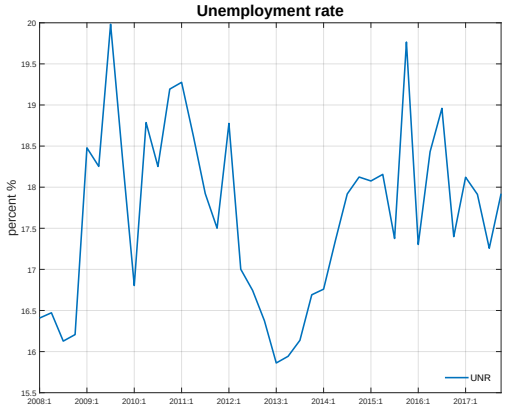
Appendix C. Graphical representation of the data



(a)

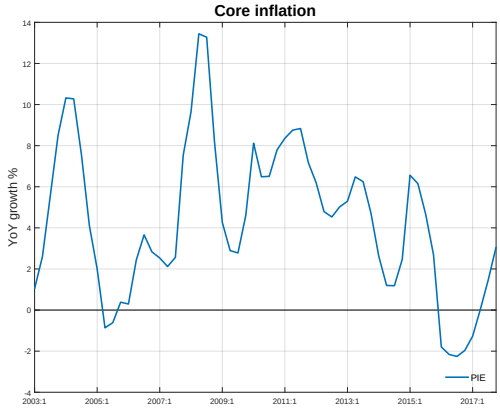


(b)

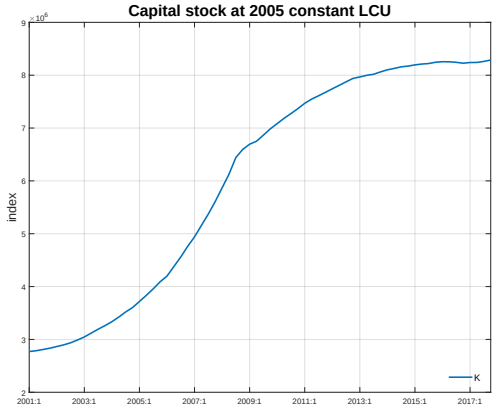


(c)

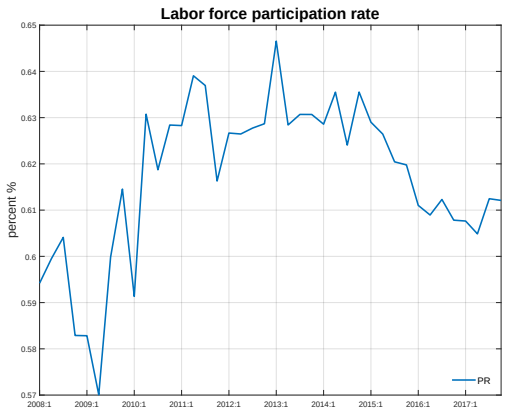
(d)



(e)



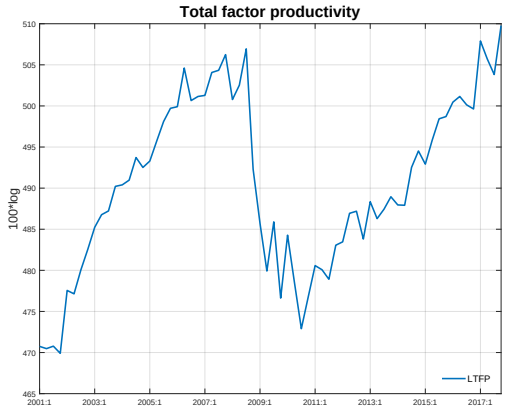
(f)


Labor force participation rate
percent %
PR
0.65
0.64
0.63
0.62
0.61
0.6
0.59
0.58
0.57
2008:1
2009:1
2010:1
2011:1
2012:1
2013:1
2014:1
2015:1
2016:1
2017:1

(g)


Labor share
alpha
0.6
0.55
0.5
0.45
0.4
0.35
0.3
2001:1
2003:1
2005:1
2007:1
2009:1
2011:1
2013:1
2015:1
2017:1

(h)


Total factor productivity
100*log
LTFP
510
505
500
495
490
485
480
475
470
465
2001:1
2003:1
2005:1
2007:1
2009:1
2011:1
2013:1
2015:1
2017:1

(i)

References

[1] Ali Alichi, Olivier Bizimana, Douglas Laxton,Kadir Tanyeri, Hou Wang, Jiaxiong Yao, and Fan Zhang (2017). Multivariate Filter Estimation of Potential Output for the United States, IMF Working Paper, WP/17/106

[2] Alichi, A., O. Bizimana, S. Domit, E. Fernndez-Corugedo, D. Laxton, K. Tanyeri, H. Wang, and F. Zhang, 2015, Multivariate Filter Estimation of Potential Output for the Euro Area and the United States, IMF Working Paper No. 15/253.

[3] Ali Alichi ; Rania A. Al-Mashat ; Hayk Avetisyan ; Jaromir Benes ; Olivier Bizimana ; Aram Butavyan ; Robert Ford ; Narek Ghazaryan ; Vahagn Grigoryan ; Mane Harutyunyan ; Anahit Hovhannisyan ; Edgar Hovhannisyan ; Hayk Karapetyan ; Mariam Kharaishvili ; Douglas Laxton ; Akaki Liqokeli ; Karolina Matikyan ; Gevorg Minasyan ; Shalva Mkhatrishvili ; Armen Nurbekyan ; Andrei Orlov ; Babken Pashinyan ; Garik Petrosyan ; Yekaterina Rezepina ; Aleksandr Shirkhanyan ; Tamta Sopromadze ; Lusine Torosyan ; Erik Vardanyan ; Hou Wang ; Jiaxiong Yao (2018). 'Estimates of Potential Output and the Neutral Rate for the U.S. Economy', IMF Working Paper No. 18/152.

[4] Benes, J., K. Clinton, R. Garcia-Saltos, M. Johnson, D. Laxton, P. Manchev, and T. Matheson, 2010, Estimating Potential Output with a Multivariate Filter, IMF Working Paper No. 10/285.

[5] Blagrave, P., R. Garcia-Saltos, D. Laxton, and F. Zhang, A Simple Multivariate Filter for Estimating Potential Output, IMF Working Paper No. 15/79.

[6] Borio, C., P. Disyatat, and M. Juselius, 2013, Rethinking Potential Output: Embedding Information about the Financial Cycle, BIS Working paper No. 404.

[7] Gran Hjelm and Kristian Jnsson , 2010, In search of a method for measuring the output gap of the Swedish economy, WORKING PAPER NR 115.

[8] Karel Havik, Kieran Mc Morrow, Fabrice Orlandi, Christophe Planas, Rafal Raciborski, Werner Rger, Alessandro Rossi, Anna Thum-Thysen, Valerie Vandermeulen, The Production Function Methodology for Calculating Potential Growth Rates and Output Gaps, European Commission, Economic Papers 535, 2014.

[9] Gali J. (2008), Monetary Policy, Inflation and the Business cycle: An introduction to the New Keynesian Framework, Princeton University Press

[10] Hodrick, R. and E. C. Prescott. 1997. Postwar U.S. Business Cycles: An Empirical Investigation. Journal of Money, Credit, and Banking, 29 (1), 116.

[11] James D. Hamilton, 2017. Why You Should Never Use the Hodrick-Prescott Filter, NBER Working Paper No. 23429

[12] Jean-Philippe Cotis, Jrgen Elmeskov, Annabelle Mourougane (2003). 'Estimates of potential output: benefits and pitfalls from a policy perspective', OECD Economics Department

[13] Kalman, R. (1960), A New Approach to Linear Filtering and Prediction Problems. Journal of Basic Engineering, Vol. 82 (1), pp. 3545

[14] K. Hagelund, F. Hansen and O. Robstad (2018), Model estimates of the output gap. Staff Memo No. 4.

[15] Laxton, D. and R. Tetlow, 1992, A Simple Multivariate Filter for the Measurement of Potential Output, Technical Report No. 59 (Ottawa: Bank of Canada).

[16] Lise Pichette, Pierre St-Amant, Ben Tomlin and Karine Anoma (2015), 'Measuring Potential Output at the Bank of Canada: The Extended Multivariate Filter and the Integrated Framework', Bank of Canada, Discussion Paper 2015-1.

[17] Marko Melolinna and Mate Toth. (2016), 'Output gaps, inflation and financial cycles in the United Kingdom', Bank of England, Working Paper No. 585.

[18] Okun, A.M., 1962, Potential GNP: Its Measurement and Significance, in Proceedings of the Business and Economic Statistics Section, pp. 98104 (Washington: American Statistical Association).

[19] Orphanides, A. and S. van Norden (2002), The unreliability of output gap estimates in real time, Review of Economics and Statistics, Vol. 84, pp. 569-583.

[20] Orphanides, A (2003), The quest for prosperity without inflation, Journal of Monetary Economics, 50(3):633-663.

[21] Igor Vetlov, Tibor Hldik, Magnus Jonsson, Henrik Kucsera and Massimiliano Pisani, (2010), Potential output in DSGE models, ECB, working paper, NO 1351