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Fitting Armenian data to the simple DSGE model with permanent productivity growth

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Abstract

This paper discusses the evaluation of structural parameters and estimated potential economic growth of Armenia using different specifications of DSGE models. We extend the simple models so that they are consistent with a balanced steady state growth path driven by deterministic labor-augmenting technological progress. Using a Bayesian likelihood approach, paper estimates DSGE models for the Armenian economy using three macro-economic time series. As a result, the dynamics of estimated potential economic growth of the model with demand and mark-up shocks is consistent with economic stylized facts contrary to other models that have no demand and markup shocks or only have one of these shocks. Additionally, estimated potential economic growth of the model with demand and markup shocks shows high correlation with other estimates of Central Bank of Armenia. Paper then structures and estimates two specifications of simple RBC model and the estimated potential economic growth of the model with persistent permanent productivity is identical with DSGE's one. We show that our models are able to beat Vector Autoregression (VAR) models in out-of-sample forecasting of economic growth.

JEL classification: C11, C32, E12, E32
Keywords: Bayesian Estimation, VAR, Real Business Cycles, DSGE

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Contents

1 Introduction

It has been widely acknowledged that estimated DSGE models are able to fit the data as well as VAR models do. These models have become a powerful tool for macroeconomic analysis. Such models are derived from microeconomic foundations and with simple monetary policy rule. DSGE models also include financial and non-financial frictions to capture properties of data, for example, investment adjustment costs, price and wage stickiness, habit in consumption etc. These models are used for the analysis of business cycle fluctuations and effects of monetary policy. DSGE models also have ability to 'tell stories' in a policy making context.

The main purpose and motivation of this paper are to construct the first balanced growth path model for Armenia, to estimate and evaluate the parameters of the Armenian economy and potential economic growth with structural models and incorporate balanced growth path in DSGE model for forecasting. For this purpose several models are constructed and estimated using Bayesian techniques for Armenian economy. We construct all the models with assumption of balanced growth path. Balanced growth path models assume that most of macroeconomic series, such as GDP, consumption, capital, real wage, investment grow at the same rate and the consumption/output and investment/output ratios do not. Stylized facts show that there are no trends in consumption/output and investment/output ratios for Armenian economy. Paper also highlights the importance of different structural shocks for getting good fit of the model.

At the first stage, the paper estimates DSGE model. To estimate the model, three macroeconomic time series of Armenia are used: GDP growth, inflation and nominal interest rate. In our DSGE model we have permanent productivity with some persistence. Permanent productivity, temporary productivity and monetary policy shocks are included in the initial specification of DSGE model. Looking at the historical evaluation of estimated potential economic growth of DSGE model, we notice strange downs and ups in some periods, which is not economically meaningful and is not consistent with historical events of Armenian economy. To get better estimated potential economic growth from DSGE model, demand and mark-up shocks are added into the model both separately and together. Looking at the values of likelihood functions, model with demand shock fits data better compared to other specifications. But the estimated potential growth of this model is similar to the first DSGE model. In the model with demand and mark-up shocks, the mode value of Calvo parameter is estimated to be high compared to other specifications. In the model with mark-up shock the standard error of mark-up shock is estimated to be approximately two times less than in the model with demand and mark-up shocks. What refers to the dynamics of different models' estimated potential economic growth, the model with demand and mark-up shocks is consistent with economic stylized facts in contrast to other models that have no demand and markup shocks or only have one of these shocks. Although the value of likelihood function of DSGE model with demand and mark-up shocks is costly relative to the model with demand shock only, its estimated potential economic growth shows a high correlation with other estimates of Central Bank of Armenia (HP filter, Multivariate Kalman filter and Production function approach). But it does not mean that we have perfectly estimated potential growth.

At the next stage, the paper estimates 2 specifications of RBC model. For the estimation of the RBC model we use data of GDP growth. The dynamics of the model is driven by permanent and temporary productivity shocks. First specification of RBC is represented by persistence in permanent technology. On the other hand, in the second specification permanent productivity is a white noise. Results show that posterior means of estimated parameters are very similar except standard errors of shocks. Dynamics of estimated potential economic growth from the model with persistent productivity is highly correlated with the one estimated from DSGE with demand and markup shocks.

The remaining of the paper is organized as follows. Section 2 reviews the literature related to the research topic. Section 3 present stylized facts. In section 4 we present DSGE model. Section 5 describes data and estimation process of the model. Section 6 compares out-of-sample forecast performance of the estimated RBC and DSGE models with econometric models. Last section concludes.

2 Literature Review

There is a large literature considering the theoretical and empirical aspects of non-stationary time series, including the random-walk process. Many empirical studies report that most macroeconomic series, such as GDP, consumption and investment, follow a non-stationary process. Furthermore, other empirical studies estimate the decomposition between the cycle and the trend components, since macroeconomic series are thought to consist of a stationary process regarded as business cycles and a non-stationary process indicating a stochastic trend or economic growth.

Kydland and Prescott (1982) present revolutionary ideas in their paper. One of these ideas, which builds on prior work by Lucas and Prescott (1971) is to combine business cycle and growth theories. It is not surprising that a paper with so many new ideas has formed the macroeconomics research agenda of the last two decades. The models that first followed Kydland and Prescott's work were referred to as 'real business cycle' models because of their emphasis on the role of real shocks, particularly technology shocks, in driving business fluctuations.

Are business cycles mainly the result of permanent shocks to productivity? King et al.(1991) provide new evidence on this question using a large class of RBC models identifying permanent productivity shocks as shocks to the common stochastic trend in output, consumption, and investment. They found a long tradition of empirical support for balanced growth in which output, investment, and consumption display positive trend growth, but the consumption/output and investment/output ratios do not.

In order to derive necessary equations, the model is first cast into stationary form. For doing that, output, investment, consumption and other non-stationary time series are scaled by the common stochastic trend. This variable defines model's balanced growth path.

Smets and Wouters (2007) assume that output, investment, and consumption in their model expand at the same rate, which reflects the assumption of a balanced-growth path. Most of the DSGE models, for instance those by Adolfson et al. (2007) and Christiano et al.(2011), follow unit root technology shock, which induces the common stochastic trend.

Adolfson et al. (2007) develop a DSGE model for an open economy and estimate it on Euro area data, using Bayesian estimation techniques. They include a stochastic unit-root technology shock in the model as a representation of common stochastic trend. This technique allows to use non detrended growth rate data in the estimation.

Lafourcade (2012) presents an estimated new-Keynesian DSGE model for the Dutch economy. The Dutch National Income and Product Accounts (NIPA) show only a part of the features of a balanced-growth path. Specifically, no real ratio between GDP and any of its components is stationary, except government spending. Nevertheless, the ratio of nominal consumption to nominal investment is stationary, as the share of labor compensation in the sum of nominal consumption and investment. These two relationships are the foundations of the restricted balanced-growth path around which they build model. Their path is 'restricted' because not all observed variables satisfy requirements of balanced-growth.

Chang et al. (2006), using Bayesian methods, estimate two types of DSGE models: the standard specification in which hours worked are stationary and the modified version with permanent labor supply shock that can generate a unit root in hours worked. They find that data support the latter specification.

Beidas-Strom and Poghosyan (2011) present and estimate a small open economy DSGE model for the Jordanian economy. They use Bayesian technique to estimate the model. To apply this methodology, they combine priors and the likelihood function to obtain the posterior distribution of structural parameters. The model exhibits a balanced growth path. They assume that in steady-state labor productivity grows at constant rate. However, we assume that productivity is subject to both transitory and permanent shocks. A permanent productivity introduces a unit root in major aggregates.

Costa et al. (2016) build and estimate a structural growth model with micro-founded specifications of trends and cycles. They setup and estimate a standard New Neoclassical Synthesis (NNS) DSGE model with a balanced growth path built on theoretically founded growth vectors. The stochastic specification of the balanced growth path allows the model to capture data and disentangle them into permanent and transitory disturbances.

Hasumi et al. (2018) incorporate the endogenous productivity growth framework of Romer (1990) into a new Keynesian DSGE model with nominal price and wage rigidities to evaluate the Japanese economy after 1980 for over three decades including the bubble burst in 1991, the Asian currency crisis in 1998 and the Lehman Brothers' failure in 2008. They match the model parameters to the Japanese economy from 1980:Q2 to 2013:Q4 and decompose the output into trend and cycle components. Using Bayesian estimation, they decompose the original time series into cyclical and trend components and compare them with those extracted by the the Hodrick-Prescott and the band-pass filters. They found that the common trend fluctuates with much greater volatility than those of both the HP and BP filters.

Fueki et al. (2016) present a Bayesian-estimated DSGE model of Japanese economy. The key feature of the model is that it takes into account persistent growth rate shock, which enables to estimate the growth of output trend without detrending data. They define the potential output as the output under flexible prices. The result is that the output gap from their measure of potential output shows better forecasting performance for inflation—in particular, at short horizons—than other measures of output gap.

Flexible price models serve as a benchmark for measuring the natural rate of output and the output gap. "Natural" levels of macroeconomic variables have a massive importance for central banks. Natural level of production and the output gap are both of high importance in monetary theory (Walsh (2010)).

Baekken (2006) develops a flexible prices version of NEMO (Norwegian Economy Model). He focuses on the natural level of production. In NEMO there are two countries, home and abroad. Model is represented by domestic and foreign prices. In the paper, flexible price model is derived by having flexible domestic prices and imported sticky prices.

Vetlov et al. (2011) provide historical estimates of potential output and output gaps on the basis of selected DSGE models developed by the European Central Bank. They use a new approach to estimate potential output which is based on NK DSGE models. It allows to estimate alternative model-based notions of potential output encompassing the level of output obtained under flexible prices and wages. Paper reports that estimated output gaps from flexible-price models can have different signs than the HP output gap in some periods.

Coenen et al. (2009) analyze a notion of flexible-price output and output gap based on the estimated NAWM developed at the ECB. The output gap notion is used in the monetary policy reaction function. The baseline flexible-price output gap is defined as the deviation of actual output from the level of output that would prevail in an environment of full nominal flexibility in goods and labor markets (Woodford (2003)). Paper's analysis shows that compared to alternative output gap measures, its estimates of the flexible-price output gap performs relatively well in predicting euro area inflation over medium-term horizons.

This paper relates to several studies. Labor productivity in the production function of intermediate goods producers is introduced following Adolfson et al. (2007). The flexible price economy's block is included in the model in a way suggested in Smets and Wouters (2007). The Calvo nonlinear Phillips curve is derived as in Sims (2014). As the model developed in this paper is estimated by Bayesian method, it also relates to studies in these field such as An and Schorfheide (2006) and Canova (2007).

3 Stylized facts

Balanced growth is defined as an long-run equilibrium path where most macroeconomic series, such as GDP, consumption, capital, real wage and investment grow at the same rate and the consumption/output, capital/output and etc. do not. If an economy is not yet on its balanced-growth path, it will tend to go toward that path. For example, if a country has a small capital stock relative to GDP, then its capital stock will grow faster than real GDP. Countries that are still developing may well be in this position. Countries that are further along in the development process are likely to be on their balanced-growth paths. For such countries, the ratio of capital stock to output is unchanging.

Figure 1 shows consumption/output and investment/output ratios for advanced economies.

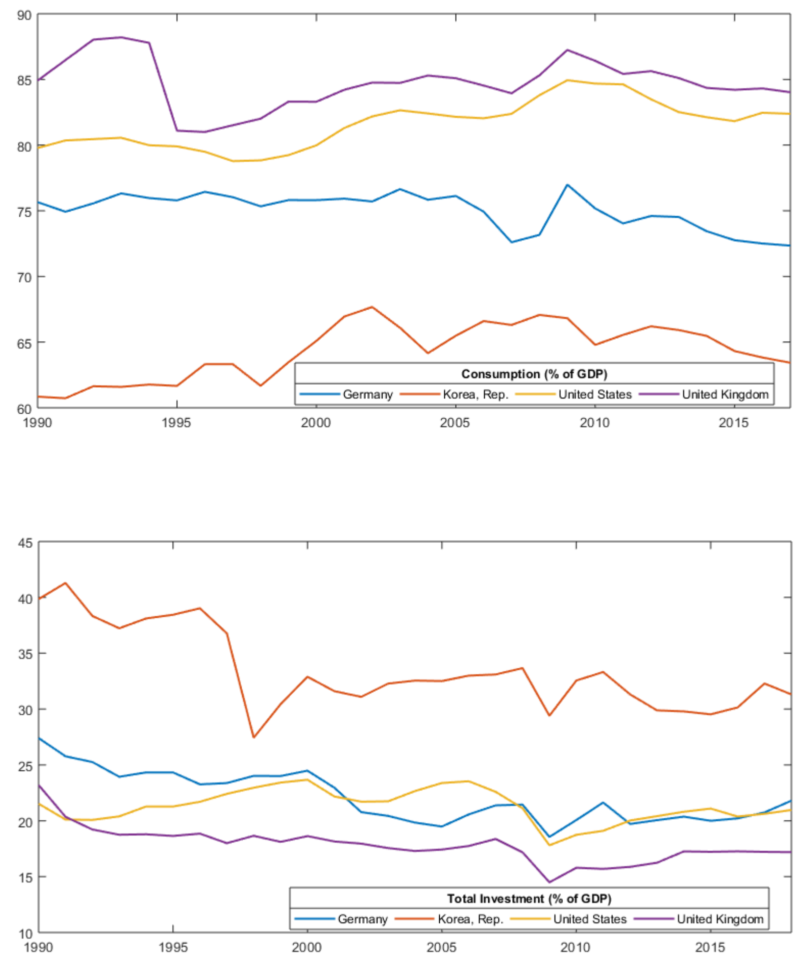

Figure 1: Consumption and Total Investment (% of GDP)

One can see from these graphs that the great ratios in advanced economies such as US, UK, Germany and Korea, Rep., are on average constant which means that these coountries are on their balanced-growth paths.

The next figure depicts great ratios for Armenian economy.

Figure 2: Consumption and Total Investment (% of GDP)

Figure 2 shows that there are no obvious trends in these ratios. Nevertheless, our data is too short to confirm the presence or absence of a trend. However, there is no reason that can prevent us from building a model with balanced-growth path assumption and estimate it with Armenian data.

4 The Model

The basic structure of the model is as follows. Households maximize their utility function subject to a budget constraint, which states that all sources of income must equal all uses of income within each period. Households are assumed to own all factors of production in the economy (capital and labor in our simple model). Because they rent out labor and capital to firms, households are paid wages and the return to capital, respectively. They also hold some amount of government bonds that pay a nominal riskless interest rate. The final output is manufactured by a final good producer, which uses a continuum of intermediate goods as inputs. The intermediate good producers rent capital and labor to manufacture their goods with the technology that is assumed to be Cobb–Douglas production function. The monetary authority sets the nominal interest rate via Taylor rule.

4.1 Households

The household maximizes the following lifetime utility function, which is separable in consumption, C_t and hours worked, N_t:

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\log(C_{t+j}) - \frac{N_{t+j}^{1+\varphi}}{1+\varphi} \right) \tag{4.1.1}$$

where E_t is the expectation operator condition on information available at time t, β is the discount factor and φ is the inverse of the Frisch elasticity of labor supply.
Household maximizes the utility function subject to the following budget constraint:

$$C_t + I_t + \frac{B_t}{P_t} = \frac{W_t N_t}{P_t} + \frac{R_t^k K_{t-1}}{P_t} + \frac{R_{t-1} B_{t-1}}{P_t} + \frac{Div_t}{P_t} \tag{4.1.2}$$

where I_t is investment, P_t the consumer price index, B_t an amount of government bonds that pay a nominal gross interest rate of R_t; W_t the nominal wage, R_t^k the rental price of capital K_{t-1} and Div_t are dividends of firms, which are owned by households. Law of motion for capital is represented by the following function:

$$K_t = (1-\delta)K_{t-1} + I_t \tag{4.1.3}$$

where δ is the depreciation rate of capital. From (4.1.3):

$$I_t = K_t - (1-\delta)K_{t-1} \tag{4.1.4}$$

Inserting (4.1.4) into (4.1.2), the budget constraint gets the form:

$$C_t + K_t - (1-\delta)K_{t-1} + \frac{B_t}{P_t} = \frac{W_t N_t}{P_t} + \frac{R_t^k K_{t-1}}{P_t} + \frac{R_{t-1} B_{t-1}}{P_t} + \frac{Div_t}{P_t} \tag{4.1.5}$$

Given our description of the household's problem, the Lagrangian function associated with it is represented by the following:

$$\underset{\{C_t,N_t,B_t,K_t\}}{\mathcal{L}} = E_t \sum_{j=0}^{\infty} \beta^j \Bigg[\bigg(\log(C_{t+j}) - \frac{N_{t+j}^{1+\varphi}}{1+\varphi} \bigg) - \lambda_{t+j} \bigg(C_{t+j} + K_{t+j} - (1-\delta)K_{t+j-1} + \frac{B_{t+j}}{P_{t+j}} - \frac{W_{t+j}N_{t+j}}{P_{t+j}} - \frac{R^k_{t+j}K_{t+j-1}}{P_{t+j}} - \frac{R_{t+j-1}B_{t+j-1}}{P_{t+j}} - \frac{Div_{t+j}}{P_{t+j}} \bigg) \Bigg] \tag{4.1.6}$$

The first order conditions with respect to C_t, N_t, B_t, K_t are:

$$C_t : \frac{1}{C_t} - \lambda_t = 0 \tag{4.1.7}$$

$$N_t : -N_t^{\varphi} + \lambda_t \frac{W_t}{P_t} = 0 \tag{4.1.8}$$

$$B_t : -\frac{\lambda_t}{P_t} + \frac{\beta \lambda_{t+1}}{P_{t+1}} R_t = 0 \tag{4.1.9}$$

$$K_t : -\lambda_t + \beta(1-\delta)\lambda_{t+1} + \frac{\beta \lambda_{t+1}}{P_{t+1}} R^k_{t+1} = 0 \tag{4.1.10}$$

Combining (4.1.7) with (4.1.9), we get consumption Euler equation. It shows the relationship between today's and tomorrow's consumption given interest rate and inflation expectations.

$$C_t = \frac{1}{\beta} C_{t+1} \Pi_{t+1} R_t^{-1} \tag{4.1.11}$$

Rewriting (4.1.8) and using the definition of Lagrangian multiplier, we are left with the household's labor supply equation:

$$\frac{W_t}{P_t} = N_t^{\varphi} C_t \tag{4.1.12}$$

From (4.1.10):

$$\frac{R^k_{t+1}}{P_{t+1}} = \frac{1}{\beta} \frac{C_{t+1}}{C_t} - (1-\delta) \tag{4.1.13}$$

which determines the capital supply equation.

Following much of the literature on DSGE models with balanced growth path (Smets and Wouters (2007), Christiano et al. (2011), Adolfson et al. (2007)), we assume that the non-stationary variables in our model grow at the same rate, which reflects the assumption of a balanced-growth path. Thus, output, investment, consumption and other non-stationary time series are scaled by the

common stochastic productivity trend. For example, $C_t = \tilde{C}_t Z_t$ where $\tilde{C}_t$ is the cyclical component, Z_t the trend component.
Stationary forms of (4.1.3) , (4.1.11) , (4.1.12) , (4.1.13) equations are:

$$\tilde{K}_t \mu_{z,t} = (1-\delta)\tilde{K}_{t-1} + \tilde{I}_t \mu_{z,t} \tag{4.1.14}$$

$$\tilde{C}_t = \frac{1}{\beta}\tilde{C}_{t+1}\Pi_{t+1}R_t^{-1}\mu_{z,t+1} \tag{4.1.15}$$

$$\frac{\tilde{W}_t}{P_t} = N_t^{\varphi}\tilde{C}_t \tag{4.1.16}$$

$$\frac{R_{t+1}^k}{P_{t+1}} = \frac{1}{\beta}\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\mu_{z,t+1} - (1-\delta) \tag{4.1.17}$$

where $\mu_{z,t} = \frac{Z_t}{Z_{t-1}}$ is the gross growth rate of technology.

(4.1.15), (4.1.16) and (4.1.17) equations determine the rational, forward-looking household's allocation decisions.

4.2 Final good producer

Final good is produced using intermediate goods with the following CES production function:

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4.2.1}$$

where ε is the elasticity of substitution between continuum of goods.

Final good producer maximizes its profit subject to the production function (4.2.1), taking as given all intermediate goods prices P_{it}:

$$P_t Y_t - \int_0^1 P_{it} Y_{it} di \to \max \tag{4.2.2}$$

Final good producer plays a role of aggregator. It buys differentiated products from intermediate goods producers and combines them into a single final good, which it sells in a perfectly competitive market. Final good producer's problem is to find demand functions for i-th intermediate good.

Inserting (4.2.1) into (4.2.2), maximization problem gets the form:

$$P_t\left(\int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_{it} Y_{it} di \to \max \tag{4.2.3}$$

The first order condition with respect to Y_{it} is:

$$Y_{it} : P_t \frac{\varepsilon}{\varepsilon-1}\left(\int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}-1} \frac{\varepsilon-1}{\varepsilon} Y_{it}^{\frac{\varepsilon-1}{\varepsilon}-1} - P_{it} = 0 \tag{4.2.4}$$

Simplifying (4.2.4), we get demand function for good i:

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t \tag{4.2.5}$$

where P_t is defined as an aggregate price index, which is represented by the following CES function.

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}} \tag{4.2.6}$$

4.3 Intermediate goods producers

There is a continuum of intermediate goods producers. Each intermediate good producer has access to a technology represented by a production function

$$Y_{it} = A_t(K_{it-1})^{\alpha}(Z_t N_{it})^{1-\alpha} \tag{4.3.1}$$

where Y_{it} is the output of i-th firm in period t, A_t the cyclical or temporary productivity, α the share of capital, $1-\alpha$ the share of labor used in production function and Z_t is a productivity of labor. We can drop the i subscript because all the firms are identical and as we can see later they choose the same labor/-capital ratio. They seek to maximize a profit by choosing the quantities of each input demanded.

$$P_t Y_t - W_t N_t - R_t^k K_{t-1} \rightarrow \max \tag{4.3.2}$$

subject to a production function. Inserting (4.3.1) into (4.3.2), we get:

$$P_t A_t K_{t-1}^{\alpha}(Z_t N_t)^{1-\alpha} - W_t N_t - R_t^k K_{t-1} \rightarrow \max \tag{4.3.3}$$

FOCs of the problem are represented by the followings:

$$K_{t-1} : \alpha P_t A_t (K_{t-1})^{\alpha-1}(Z_t N_t)^{1-\alpha} - R_t^k = 0 \tag{4.3.4}$$

$$N_t : (1-\alpha) P_t A_t (K_{t-1})^{\alpha}(Z_t)^{1-\alpha}(N_t)^{-\alpha} - W_t = 0 \tag{4.3.5}$$

Rewriting (4.3.4) and (4.3.5), demand equations for labor and capital gets the following forms.

$$\frac{R_t^k}{P_t} = \frac{\alpha \tilde{Y}_t}{\tilde{K}_{t-1}} \mu_{z,t} \tag{4.3.6}$$

$$\frac{\tilde{W}_t}{P_t} = \frac{(1-\alpha)\tilde{Y}_t}{N_t} \tag{4.3.7}$$

Combining (4.3.6) with (4.3.7):

$$\frac{R_t^k}{\tilde{W}_t} = \frac{\alpha}{1-\alpha}\left(\frac{N_t}{\tilde{K}_{t-1}}\right)\mu_{z,t} \tag{4.3.8}$$

This is an important equation. As we say above, this equation shows that all the firms must have the same labor/capital ratio. (4.3.8) formula shows that the labor/capital ratio is an inverse function of $\frac{\tilde{W}_t}{R_t^k}$ relative price of production factors.

Because firms face the same factor prices and hire capital and labor in the same ratio and all face the same productivity shock, we can state that they have the same marginal costs. Thus, using (4.3.6), (4.3.7) and $P_t = MC_t$ condition, we derive marginal cost (Villaverde et. al (2006), Costa Junior et. al (2018)).

$$MC_t = \left(\frac{R_t^k}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \frac{1}{A_t Z_t^{1-\alpha}} \tag{4.3.9}$$

Decomposing non-stationary variables into cyclical and trend components, marginal cost equation gets the form.

$$MC_t = \left(\frac{R_t^k}{\alpha}\right)^{\alpha} \left(\frac{\tilde{W}_t}{1-\alpha}\right)^{1-\alpha} \frac{1}{A_t} \tag{4.3.10}$$

The remaining part of the subsection considers pricing decision. Following Calvo (1983), in each period, a fraction $1-\theta$ of firms can change their prices. This means that the probability that a firm will be stuck with the same price one period is θ, θ^2 is for two periods, and so on. Since there is a chance that the firm will get stuck with its price for multiple periods, the pricing problem becomes dynamic. The problem of the firms is then represented by:

$$\max_{P_{it}} E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[\left(\frac{P_{it}}{P_{t+j}}\right) Y_{it+j} - MC_{t+j} Y_{it+j} \right] \tag{4.3.11}$$

subject to the demand function:

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t \tag{4.3.12}$$

where $Q_{t,t+j}$ is the stochastic discount factor and equals to:

$$Q_{t,t+j} = \beta^j \left(\frac{Y_t}{Y_{t+j}}\right) \tag{4.3.13}$$

Solving (4.3.11), we find the optimal price P_t^*:

$$P_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j Y_t P_{t+j}^{\varepsilon} MC_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j Y_t P_{t+j}^{\varepsilon-1}} \tag{4.3.14}$$

We can write (4.3.14) in a more compact way as follows:

$$P_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{X_{1,t}}{X_{2,t}} \tag{4.3.15}$$

where

$$X_{1,t} = E_t \sum_{j=0}^{\infty} \theta^j \beta^j Y_t P_{t+j}^{\varepsilon} MC_{t+j} \tag{4.3.16}$$

$$X_{2,t} = E_t \sum_{j=0}^{\infty} \theta^j \beta^j Y_t P_{t+j}^{\varepsilon-1} \tag{4.3.17}$$

are auxiliary variables to make the equation convenient.

From (4.3.16) and (4.3.17) recursive formulas for $X_{1,t}$ and $X_{2,t}$ get the following forms:

$$X_{1,t} = Y_t P_t^{\varepsilon} MC_t + \theta\beta X_{1,t+1} \tag{4.3.18}$$

$$X_{2,t} = Y_t P_t^{\varepsilon-1} + \theta\beta X_{2,t+1} \tag{4.3.19}$$

Now we need to cast these equations into stationary forms. Thus, two new auxiliary variables are defined as follows:

$$x_{1,t} = \frac{X_{1,t}}{P_t^{\varepsilon}} \tag{4.3.20}$$

$$x_{2,t} = \frac{X_{2,t}}{P_t^{\varepsilon-1}} \tag{4.3.21}$$

Multiplying and dividing terms in period $t+1$ by P_{t+1}^{ε} and expressing in inflation terms, (4.3.20) and (4.3.21) equations get the following forms:

$$x_{1,t} = Y_t MC_t + \theta\beta \Pi_{t+1}^{\varepsilon} x_{1,t+1} \mu_{z,t+1} \tag{4.3.22}$$

$$x_{2,t} = Y_t + \theta\beta \Pi_{t+1}^{\varepsilon-1} x_{2,t+1} \mu_{z,t+1} \tag{4.3.23}$$

Since we have $X_{1,t}$ divided by P_t^{ε} and $X_{2,t}$ by $P_t^{\varepsilon-1}$, then (4.3.15) becomes:

$$P_t^* = \frac{\varepsilon}{\varepsilon-1} P_t \frac{x_{1,t}}{x_{2,t}} \tag{4.3.24}$$

Dividing both sides by P_{t-1}, we can express (4.3.24) with inflation terms:

$$\Pi_t^* = \frac{\varepsilon}{\varepsilon-1} \Pi_t \frac{x_{1,t}}{x_{2,t}} \tag{4.3.25}$$

Because of the fact that only $1-\theta$ fraction of firms changes price every period, the aggregate price level gets the form:

$$P_t^{1-\varepsilon} = (1-\theta)(P_t^*)^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \tag{4.3.26}$$

Dividing both sides by $P_{t-1}^{1-\varepsilon}$, (4.3.26) becomes:

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\Pi_t^*\right)^{1-\varepsilon} \tag{4.3.27}$$

Equation (4.3.27) makes it clear that inflation results from the fact that firms re-optimizing in any given period choose a price that differs from the economy's average price in the previous period.

The set of equations in (4.3.22), (4.3.23), (4.3.25) and (4.3.27) compose the Calvo nonlinear Phillips curve.

4.4 Market clearing condition

In equilibrium firms and households optimally choose prices with respect to their constraints and each market clears. The market clearing requires that the aggregate demand equals to aggregate output in the economy. Market clearing implies that aggregate bond holdings are zero because any dollar borrowed must be lent by somebody.

Household gets wage payments for working and dividends from firms. Wage payments and dividends is spent on consumption. So, from household's budget constraint we can write:

$$C_t + I_t = \frac{W_t N_t}{P_t} + \frac{R_t^k K_{t-1}}{P_t} + \frac{Div_t}{P_t} \tag{4.4.1}$$

On the other hand, since the household owns firms , their dividends are equal to:

$$\frac{Div_t}{P_t} = Y_t - \frac{W_t N_t}{P_t} - \frac{R_t^k K_{t-1}}{P_t} \tag{4.4.2}$$

Combining (4.4.1) and (4.4.2), we get market clearing condition of the form:

$$Y_t = C_t + I_t \tag{4.4.3}$$

which shows that produced output is divided into consumption and investment.

4.5 Flexible price economy

Flexible price models serve as a benchmark for measuring the natural rate of output. Flexible price economy's block is used in Smets and Wouters (2007), Edge et al. (2007) and in Justiniano and Primiceri (2008). They define the output gap as the difference of the actual output level and the output level that would prevail under flexible prices. In terms of modeling, a flexible price economy's block has been added to the original model as a parallel economy in which prices are fully flexible and markets are perfectly competitive. Superscript f is used to denote that variables are in the flexible price economy.

When there is no price stickiness $\theta = 0$, we have that $\Pi_t^* = \Pi_t$. Using this condition, we can derive the marginal cost in flexible price economy. To do that, we insert (4.3.22) and (4.3.23) into (4.3.25):

$$\Pi_t^* = \frac{\varepsilon}{\varepsilon - 1} \Pi_t \frac{Y_t MC_t + \theta\beta\Pi_{t+1}^{\varepsilon} x_{1,t+1}}{Y_t + \theta\beta\Pi_{t+1}^{\varepsilon-1} x_{2,t+1}} \tag{4.5.1}$$

Knowing that in flexible price economy $\theta = 0$ and $\Pi_t^* = \Pi_t$, we derive the marginal cost in flexible price economy, which gets the form.

$$MC_t^f = \frac{\varepsilon - 1}{\varepsilon} \tag{4.5.2}$$

In the case with flexible prices optimal price equation becomes:

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} MC_t \tag{4.5.3}$$

Then inserting (4.5.2) into (4.5.3), we get:

$$P_t^* = P = 1 \tag{4.5.4}$$

As we mention above, the market is perfectly competitive which means that $\varepsilon \to \infty$:

$$MC_t^f = \lim_{\varepsilon \to \infty} \frac{\varepsilon - 1}{\varepsilon} = 1 \tag{4.5.5}$$

Taking into account the original model's equations, flexible price block is represented by the set of following equations:

$$\tilde{K_t}^f \mu_{z,t} = (1-\delta)\tilde{K}_{t-1}^f + \tilde{I_t^f} \mu_{z,t} \tag{4.5.6}$$

$$\tilde{C_t^f} = \frac{1}{\beta} \tilde{C}_{t+1}^f \big(R_t^f\big)^{-1} \mu_{z,t+1} \tag{4.5.7}$$

$$\frac{\tilde{W_t^f}}{P} = \big(N_t^f\big)^{\varphi} \tilde{C_t}^f \tag{4.5.8}$$

$$\tilde{Y_t}^f = A_t \big(\tilde{K}_{t-1}^f\big)^{\alpha} \big(N_t^f\big)^{1-\alpha} \mu_{z,t}^{-\alpha} \tag{4.5.9}$$

$$\frac{\big(R_{t+1}^f\big)^k}{P} = \frac{1}{\beta} \frac{\tilde{C}_{t+1}^f}{\tilde{C_t^f}} \mu_{z,t+1} - (1-\delta) \tag{4.5.10}$$

$$\frac{R_t^{fk}}{\tilde{W}_t^f} = \frac{\alpha}{1-\alpha} \left(\frac{N_t^f}{\tilde{K}_{t-1}^f} \right) \mu_{z,t} \tag{4.5.11}$$

$$A_t = \left(\frac{R_t^{fk}}{\alpha} \right)^{\alpha} \left(\frac{\tilde{W_t^f}}{1-\alpha} \right)^{1-\alpha} \tag{4.5.12}$$

$$Y_t^f = C_t^f + I_t^f \tag{4.5.13}$$

From flexible price block, we get the natural output Y_t^f, which is the output under flexible prices. Thus, output gap is given by the following.

$$Y_t^{gap} = \frac{Y_t}{Y_t^f} \tag{4.5.14}$$

4.6 Monetary Policy

To complete the model, the endogenous interest rate must be set by the monetary authority. We assume that the monetary authority sets the short-term nominal interest rate following a Taylor-type feedback rule:

$$\frac{R_t}{R^{ss}} = \left(\frac{R_{t-1}}{R^{ss}}\right)^{\rho} \left[\left(\frac{\Pi_{t+1}}{\Pi^{ss}}\right)^{\phi_\pi} \left(Y_t^{gap}\right)^{\phi_y}\right]^{1-\rho} \varepsilon_t^{mp} \tag{4.6.1}$$

where $0 < \rho < 1$ is the degree of interest rate smoothing, ϕ_π, ϕ_y are responses to inflation deviation from target and output gap, ε_t^{mp} is the monetary policy shock and R^{ss}, Π^{ss} are steady state values of corresponding variables.

Appendix 8.1 discusses the calculation of the steady state values of all the endogenous variables in the model.

5 Estimation

This section estimates the set of DSGE models using Bayesian techniques. As usually done in the literature, some of the parameters are calibrated to match the important stylized facts in the data, but some are estimated. The estimation process of models consists of two stages:

1. The modes are computed using Chris Sims's optimization routine, which randomly changes the search direction if it reaches a cliff caused by indeterminacy or nonexistence.

2. MCMC technique is applied to get the posterior means and distributions for estimated parameters. The RW Metropolis-Hastings algorithm is used in this paper, which belongs to the general MCMC class of algorithms. It approximates a posterior distribution (i.e. creates a histogram) comprised of any combination of prior probabilities and sampling models.

First specification of DSGE model has 3 shocks:

1. Permanent productivity shock:

$$\mu_{z,t} = \rho_z \mu_{z,t-1} + (1-\rho_z)\mu_z^{ss} + \boldsymbol{\varepsilon_{z,t}} \tag{5.0.1}$$

2. Temporary productivity shock:

$$a_t = \rho_a a_{t-1} + \boldsymbol{\varepsilon_a} \tag{5.0.2}$$

3. Monetary policy shock:

$$\frac{R_t}{R^{ss}} = \left(\frac{R_{t-1}}{R^{ss}}\right)^{\rho} \left[\left(\frac{\Pi_{t+1}}{\Pi^{ss}}\right)^{\phi_\pi} \left(Y_t^{gap}\right)^{\phi_y}\right]^{1-\rho} \boldsymbol{\varepsilon_t^{mp}} \tag{5.0.3}$$

5.1 Data

To estimate the above developed model, three macroeconomic time series of Armenia are used: GDP growth, inflation and nominal interest rate. Time series start from the second quarter of 2000 and end in the fourth quarter of 2018. All series, except the interest rate, are seasonally adjusted using Census X12 algorithm. Then, GDP growth and Inflation series are divided by 100 and plus one to have time series expressed in gross terms. The annualized nominal rate is changed to a quarterly basis by dividing it by 400. These series are used as observable variables in estimation process of the models. The input data is presented in the Figure 5 (see Appendix 8.2).

Data is linked to the model as follows.

1. GDP growth.
$$Y_t^{obs} = \frac{\tilde{Y}_t}{\tilde{Y}_{t-1}} \mu_{z,t} \tag{5.1.1}$$
where $\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}}$ is the cyclical component of GDP, $\mu_{z,t}$ is the trend component of GDP.

2. Inflation
$$\pi_t^{obs} = \pi_t + \pi_{ss}^{obs} \tag{5.1.2}$$
where π_t in steady state equals to zero. The observable equation removes the inflation target and links observed inflation to the model variable.

3. Nominal interest rate
$$i_t^{obs} - i_{ss}^{obs} = i_t - ln(i_{ss}) \tag{5.1.3}$$

The steady state of observable nominal interest rate i_{ss}^{obs} is different from model calculated steady state i_{ss}. That's why the constant is added to the measurement equation displaying the difference between these variables. The steady state of i_t^{obs} is calculated having the average of the observed sample (about 8 % at an annual frequency). The value of i_t in steady state is determined from Euler equation (about 14 % at an annual frequency):

$$C^{ss} = \frac{1}{\beta} C^{ss} \Pi^{ss} \left(R^{ss}\right)^{-1} \mu_z^{ss} \tag{5.1.4}$$

$$R^{ss} = \frac{1}{\beta} \Pi^{ss} \mu_z^{ss} \tag{5.1.5}$$

Increasing the value of β, we can equalize two steady state values, but it will make households very forward looking.

5.2 Calibration

As we say above, some of the parameters are calibrated to match the important stylized facts in the data, but some are estimated. The discount factor β is set at 0.99 to imply around 4 percent real interest rate annually. The steady state of gross growth rate of technology μ_z^{ss} is 1.01535477, which corresponds to the average economic growth of Armenia from the second quarter of 2000 to the fourth quarter of 2018 (1.53 percent quarterly or 6.12 percent annually). The share of capital in production function α is 0.4. The elasticity of substitution between goods ε is set at 6 implying 20 percent markup in steady state.

5.3 Prior distribution

Remaining parameters are estimated. Beta distribution is used for the parameters that take values between zero and one, the gamma distribution for parameters that are restricted to be positive and the inverse gamma distribution for the shocks' standard errors. Labor supply elasticity φ follows gamma distribution with 1.5 mean and 0.3 standard deviation, beta distribution is given for capital depreciation rate δ with 0.03 mean (12 percent at an annual frequency) and 0.01 standard deviation. Persistence parameters of shocks ρ_a and ρ_z follow beta distribution: ρ_a with 0.7 mean and 0.15 standard deviation, ρ_z with 0.6 mean and 0.15 standard deviation. We give the inverse gamma distribution to the standard errors of ε_a and ε_z with the same 3.5 mean and infinite standard deviation to see which one is more volatile. Beta distribution is given for the smoothing parameter in Taylor rule ρ with 0.6 mean and 0.15 standard deviation. The prior mean of Calvo parameter θ is set to 0.65 with 0.15 standard deviation. Parameters of Taylor rule ϕ_π and ϕ_y follow gamma distribution: ϕ_π with 1.5 mean and 0.15 standard deviation, ϕ_y with 0.6 mean and 0.15 standard deviation. We give the inverse gamma distribution to the standard error of ε_{mp} (monetary policy shock) with 0.25 mean and infinite standard deviation. These priors are commonly used values in DSGE literature, for example Smets and Wouters (2007), Sims (2014), etc.

5.4 Parameter estimates

Posterior distributions of parameters are obtained by running 2 parallel chains of Metropolis-Hastings algorithm with 500000 draws. Acceptance rates are 25.0394% and 25.0758%. The multivariate convergence diagnostic of the model's likelihood function is stored in Appendix 8.2. After running Bayesian technique, we get estimation results of the model. Table 1 presents modes of parameters, the likelihood function value and parameters' posterior means.

	Description	Prior dist.	Mean	SD	DSGE mode	DSGE posterior mean
φ	Labor supply elasticity	Gamma	1.5	0.3	1.5231	1.5855
δ	Depreciation rate of capital	Beta	0.03	0.01	0.0404	0.0451
θ	Calvo parameter	Beta	0.65	0.15	0.5563	0.5584
ρ	Interest rate persistence	Beta	0.6	0.15	0.7152	0.7077
ϕ_π	Reaction to inflation	Gamma	1.5	0.15	1.3995	1.4192
ϕ_y	Reaction to output gap	Gamma	0.5	0.10	0.8635	0.9021
ρ_a	Persistence of temporary productivity	Beta	0.7	0.15	0.8365	0.8049
ρ_z	Persistence of permanent productivity	Beta	0.6	0.15	0.6593	0.5688
ε_a	Temporary prod. shock	Inverse Gamma	3.5	Inf	3.5266	3.5868
ε_z	Permanent prod. shock	Inverse Gamma	3.5	Inf	1.4210	1.8266
ε_{mp}	Monetary policy shock	Inverse Gamma	0.25	Inf	0.4810	0.5056
	The value of likelihood				**-618.11**	

Table 1: Modes, prior and posterior means of structural parameters

Estimation results show, that posterior means are different from their prior means. The posterior mean of δ is estimated to be 0.0449 (about 18 percent at an annual frequency). The posterior mean of θ is 0.5584, which corresponds to price duration of 2.26 quarters. The posterior mean of the inverse of labor supply elasticity is 1.5855 which means that the workers in Armenia increase labor supply less with respect to one percent increase of real wage than we priory thought. So we can verify that used data set is quite informative for the estimation of most model's parameters.

Figure 3 shows the estimated potential economic growth from DSGE model[1].

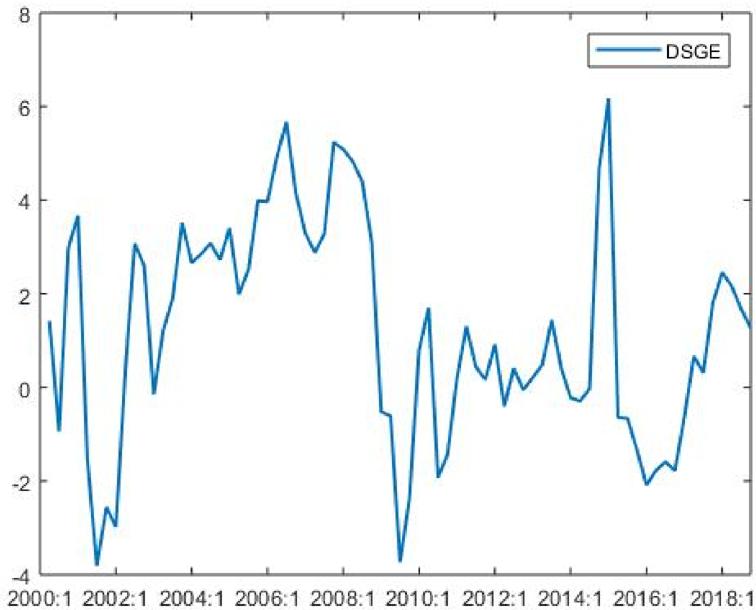

Figure 3: Estimated potential economic growth

Looking at the dynamics of estimated potential economic growth from the DSGE model, we can see strange downs and ups in periods 2001Q1-2002Q1 and 2014Q3-2016Q4, which is not economically meaningful. This could be a result of absence of demand and mark-up shocks. To get better estimated potential growth from DSGE model, two shocks are added in the model. Consumption preference shock is added into the consumption Euler equation following Smets and Wouters (2007). The shock is included both in sticky and flexible price blocks of the model:

$$\tilde{C}_t = \frac{1}{\beta}\tilde{C}_{t+1}\Pi_{t+1}R_t^{-1}\mu_{z,t+1}\boldsymbol{d_t} \tag{5.4.1}$$

$$\tilde{C}_t^f = \frac{1}{\beta}\tilde{C}_{t+1}^f\left(R_t^f\right)^{-1}\mu_{z,t+1}\boldsymbol{d_t} \tag{5.4.2}$$

where demand shock follows AR(1) process.

$$d_t = \rho_d d_{t-1} + (1-\rho_d)d^{ss} + \varepsilon_t \tag{5.4.3}$$

Next stage adds the price mark-up shock into the model in the marginal cost equation. Headline inflation is very volatile in Armenia because of seasonal

[1]Estimated potential growths are very similar in both mode and posterior mean cases.

goods. This volatility is not created by optimal behavior of price setting firms, but follows some exogenous process. Price mark-up is included in marginal cost equation of sticky model as follows:

$$MC_t = \left(\frac{R_t^k}{\alpha}\right)^{\alpha}\left(\frac{\tilde{W}_t}{1-\alpha}\right)^{1-\alpha}\frac{1}{A_t}\boldsymbol{e_{\pi,t}} \tag{5.4.4}$$

where mark-up follows the following process.

$$e_{\pi,t} = \rho_e e_{\pi,t-1} + (1-\rho_e)e_{\pi}^{ss} + \varepsilon_{\pi,t} \tag{5.4.5}$$

Last specification of DSGE model estimates the model with both demand and mark-up shocks. Table 2 shows results of the estimated modes.

	Prior Mean	DSGE	DSGE with demand shock	DSGE with mark-up shock	DSGE with demand and and mark-up shocks
φ	1.5	1.5231	1.5037	1.3588	1.3710
δ	0.03	0.0404	0.0500	0.0454	0.0350
θ	0.65	0.5563	0.5992	0.6887	0.7867
ρ	0.6	0.7152	0.8213	0.6663	0.7839
ϕ_π	1.5	1.3995	1.3225	1.4374	1.4261
ϕ_y	0.5	0.8635	1.0114	0.7110	1.0784
ρ_a	0.7	0.8365	0.8866	0.8508	0.7840
ρ_z	0.6	0.6593	0.5944	0.7947	0.6062
ρ_d	0.75	-	0.4206	-	0.8220
ρ_e	0.5	-	-	0.0762	0.1015
ε_a	3.5	3.5266	3.6583	2.8047	2.5551
ε_z	3.5	1.4210	1.6685	1.2306	1.4395
ε_{mp}	0.25	0.4810	0.4962	0.1282	0.1261
ε_d	3.5	-	0.7904	-	0.6176
ε_π	6.5	-	-	12.225	23.0134
	The value of Likelihood	**-618.11**	**-607.56**	**-627.33**	**-627.21**

Table 2: Estimated modes for 4 specifications of DSGE model

Model with consumption preference shock (DSGE with demand shock) improves the likelihood (from -618.11 to -607.56). The likelihood function values of the model with mark-up shock and the model with demand and mark-up shocks are almost identical (-627.33 and -627.21 respectively) and costly compared to the model with demand shock. In the model with demand and mark-up shocks, the mode value of Calvo parameter is estimated to be 0.7867, which makes the less volatile part of prices to be more sticky. The mode value of δ is 0.035 (about 14 percent at an annual frequency). The inverse of labor supply elasticity ϕ is 1.3710 which means that the workers in Armenia increase labor supply more with respect to one percent increase of real wages than we priory think. The temporary productivity and demand shock processes are the most persistent with an AR(1) coefficient of 0.7840 and 0.8220 respectively. In contrast, the persistence of the permanent productivity is relatively low (0.6062). The high persistence of the temporary productivity and demand processes implies that at long horizons most of the forecast error variance of the real variables is explained by those two shocks. The modes of the standard errors of the temporary productivity, permanent productivity, monetary policy, demand and mark-up shocks are 2.5551, 1.4395, 0.1261, 0.6176 and 23.0134 respectively. In the model with mark-up shock the standard error of mark-up shock is estimated to be 12.225. In contrast, the standard error of mark-up shock is 23.0134 in the model with demand and mark-up shocks. The explanation of this is the following: since in the last model the price stickiness is high (0.7867), we need more volatile mark-up shock to explain the dynamics of the same inflation row than in the model with mark-up shock. Finally, turning to the monetary policy reaction function parameters, we can say that there is a considerable degree of interest rate smoothing ρ as the mode value is 0.7839. The mode of the reaction coefficient to inflation is estimated to be relatively high (1.4261). Policy reacts strongly to the output gap contrary to our prior beliefs.

Figure 4 compares the estimated potential economic growth for all the specifications of DSGE model.[2] As demonstrated in the figure, the estimated potential economic growth has a significant jump in period 2014Q4-2015Q1 except the one from DSGE model with demand and markup shocks. Armenian economy experienced demand and markup shocks. Not having these shocks in the model, estimation gives huge increase of potential growth in that period, which is not correct from view point of historical economic events. Although the likelihood function value of DSGE model with demand and mark-up shocks is costly relative to the model with demand shock, the dynamics of it's potential economic growth is consistent with other estimates of Central Bank of Armenia[3] (HP filter, Multivariate Kalman filter and Production function approach). The correlations between them are respectively 0.79, 0.75 and 0.86. The estimated potential growth for the period before 2008 crisis is 2.4 percent quarterly, while it is dropped to 1.2 after the crisis because of structural break in the economy.

Table 7 (see Appendix 8.2) captures forecast error variance decomposition of economic growth for all the specifications of DSGE model. First specification (DSGE) shows that both in the short and long run about 85% variation of economic growth is explained by temporary productivity and monetary policy shocks. The contribution of permanent productivity shock is relatively low (about 14%). In the model with demand shock the contribution of temporary

[2]We also estimate 4 specifications of DSGE models setting $\varepsilon = 6$ in flexible price economy block. Figure 19 in Appendix 8.2 shows that in the case of $\varepsilon = 6$ the potential ecomomic growth of DSGE model with demand and markup shocks has the same dynamics as well as the model in the case of $\varepsilon \to \infty$ (the correlation between them is 0.99). All the estimation results of models with $\varepsilon = 6$ in flexible model economy are available upon request.

[3]See Central Bank of Armenia's Monetary Policy Program, Q4, 2018, pages 20-21. https://www.cba.am/EN/pperiodicals/gnach_IV_18.pdf

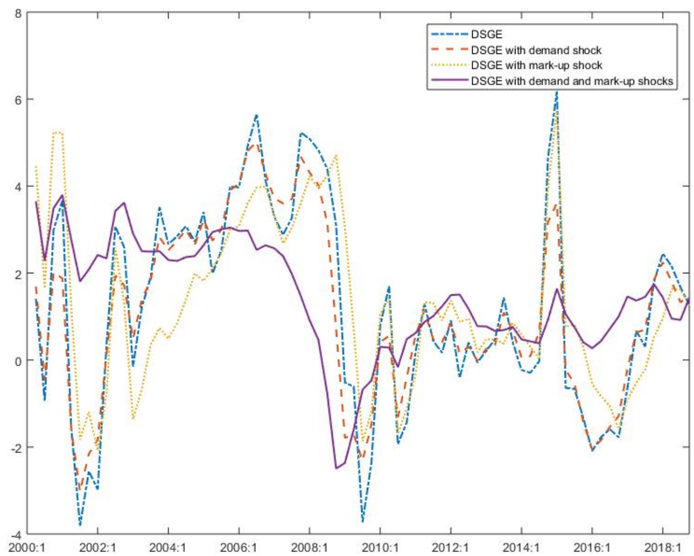

Figure 4: Potential economic growth

productivity shock decreases over time (from 62.79% to 52.06%). Permanent productivity shock drives 15.92% of economic growth variation in short run and its contribution decreases to 13.7% during longer horizons. In the next model (DSGE with mark-up shock) the contribution of mark-up shock is very low in all the periods(from 0.01% to 0.31%). Temporary productivity shock and permanent productivity shock explain about 96% variation of economic growth in the short run, which decreases to 94% in the long run. In the last model (DSGE with demand and mark-up shocks), driving forces of economic growth are temporary productivity and demand shocks (about 82%). Temporary productivity shock decreases over time (from 51% to 41%) in contrast to demand shock (from 30% to 40%). The contribution of mark-up shock and monetary policy shock is low over time (about 5%). Permanent productivity decreases over time (from 14.69% to 11.93%).

Figures 14-17 (see Appendix 8.2) capture historical decomposition of economic growth for all DSGE models. Besides the permanent productivity shock, other shocks affect the level of GDP's cyclical component, which makes us very uncomfortable to discuss the decomposition of GDP growth, but we can do some conclusions looking at the graphs. Figure 14 captures the historical decomposition of economic growth from estimated DSGE model. As expected, the dynamics of permanent productivity shock isn't smooth as much and makes economic growth to be very volatile with respect to fundamental productivity. Demand shock (see Figure 15) takes on itself some parts of the temporary productivity and permanent productivity shocks. As one can see from Figure 16, the mark-up shock has almost no contribution in the decomposition of economic growth. Figure 17 displays historical decomposition of economic growth for the model with both demand and mark-up shocks. The historical contribution of permanent productivity growth on GDP growth is less volatile and captures historical developments in Armenia.

Estimated structural shocks of the model are stored in Appendix 8.2 (see Figure 18). Global financial crisis is captured by negative permanent and temporary productivity shocks, as well as by negative demand shock. Negative temporary productivity shocks in 2010 are the result of bad weather conditions, which led to a huge decline of the agriculture sector. Since the fourth quarter of 2014, remittances flowing to Armenia declined significantly, which was a result of Russian crisis. Armenian risk premium increased and generated higher inflation expectations. As a result, consumers started to spend much, which is captured by the estimated positive demand shock for that period. Central Bank of Armenia tightened monetary conditions significantly to break higher inflation expectations. After stabilizing inflation expectations, estimated model generates positive monetary policy shock for the mentioned period. The mark-up shock is very volatile, because the inflation of agriculture goods is very unstable. Overall, the estimated structural shocks of the model are supported by stylized facts and historical events.

The rest of this section estimates two specifications of RBC model to compare its estimated potential growth with DSGE estimates. RBC is a "Real" model because there is no role for nominal variables. As a result, monetary policy has no impact in the model. Moreover, markets are perfectly competitive in RBC framework. So, we can state, that the RBC model is identical with the flexible price economy's block in our DSGE model. There are two shocks in RBC model: permanent productivity ($\mu_{z,t}$) shock and temporary productivity (a_t) shock. The following two cases are discussed.

1. the permanent productivity with some persistence identical to its specification in DSGE model:

$$\mu_{z,t} = \rho_z \mu_{z,t-1} + (1-\rho_z)\mu_z^{ss} + \varepsilon_{z,t} \tag{5.4.6}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_a \tag{5.4.7}$$

2. the permanent productivity without persistence:

$$\mu_{z,t} = \mu_z^{ss} + \varepsilon_{z,t} \tag{5.4.8}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_a \tag{5.4.9}$$

Table 3 presents modes of parameters. The mode check plots are stored in Appendix 8.2 (see Figures 20 and 21). Results imply that the mode of δ decreases in both cases (from 12 percent to approximately 10 percent at an annual frequency). In RBC model with persistent productivity ρ_a is smaller than in RBC model without persistent productivity, but the standard error of temporary productivity shock is bigger in the first one. Likelihood values show, that the model with persistent productivity is a little bit costly in fitting data compared to another estimated model. But the difference in likelihoods is around 2, which is not so significant.

	Description	Prior dist.	Mean	SD	RBC with persistent prod.(mode)	RBC without persistent prod.(mode)
φ	Labor supply elasticity	Gamma	1.5	0.3	1.4229	1.4285
δ	Depreciation rate of capital	Beta	0.03	0.01	0.0269	0.0274
α	Share of capital in production	Beta	0.4	0.12	0.3695	0.3330
ρ_a	Persistence of temporary prod.	Beta	0.7	0.15	0.6292	0.7511
ρ_z	Persistence of permanent prod.	Beta	0.6	0.15	0.6753	-
ε_a	Temporary productivity shock	Inverse Gamma	3.5	Inf	2.3948	1.7171
ε_z	Permanent productivity shock	Inverse Gamma	3.5	Inf	1.5929	3.7603
	The value of Likelihood				**-143.97**	**-141.96**

Table 3: Modes of structural parameters

Figure 5 shows the estimated potential economic growth of above 2 specifications of RBC.

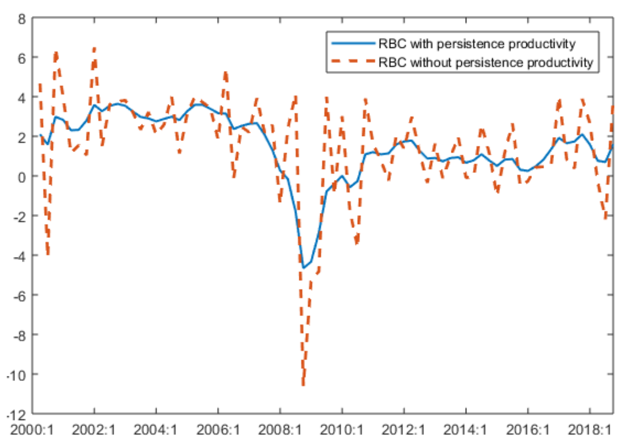

Figure 5: Estimated potential economic growth from RBC

One can verify from the graph that the potential economic growth of the model with persistent productivity is not so volatile. Although the likelihood function value of RBC model with persistent productivity is a little costly, the estimated structural potential economic growth has the same dynamics as well as the potential economic growth of DSGE model with demand and markup shocks. The correlation between them is approximately 0.94. The standard deviations of these rows are 1.65 for RBC model and 1.31 for DSGE's one, which shows that the estimated potential growth of DSGE model is much smoother. But having estimated identical (highly correlated) RBC potential growth with DSGE's one, it does not mean that we have perfectly estimated potential growth.

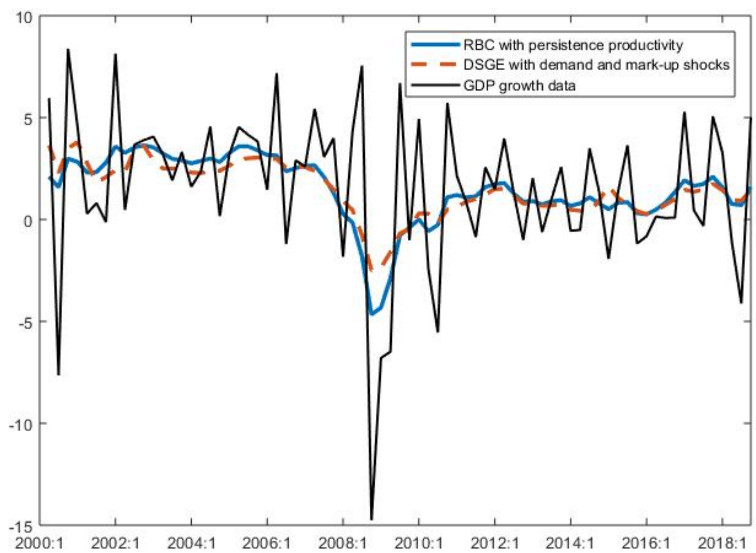

Figure 6: Comparison of estimated potential growth

At the next stage, we run the MCMC to get the posterior means and distributions of estimated parameters. Parameters are obtained from running 2 parallel chains of Metropolis-Hastings algorithm with 500000 draws. Acceptance rates are 28.281% and 28.7084% for the model with persistent productivity and 26.235% and 26.0682% for the another one. The multivariate convergence diagnostic of the models' likelihood functions are presented in Appendix 8.2. The convergence is achieved when two lines are stabilized horizontally and should be close to each other.

After running the MCMC algorithm it can be seen from the Table 4 that the posterior means of the parameters are not so different from their prior means. Differences are visible only in parameters associated with shocks.

	Prior mean	RBC with persistent prod.(mode)	RBC without persistent prod.(mode)	RBC with persistent prod.(mcmc)	RBC without persistent prod.(mcmc)
φ	1.5	1.4229	1.4285	1.4978	1.5067
δ	0.03	0.0269	0.0274	0.0302	0.0309
α	0.4	0.3695	0.3330	0.4034	0.3836
ρ_a	0.7	0.6292	0.7511	0.6573	0.7537
ρ_z	0.6	0.6753	-	0.5953	-
ε_a	3.5	2.3948	1.7171	2.5121	2.0133
ε_z	3.5	1.5929	3.7603	2.0546	3.7707

Table 4: Priors, modes and posterior means of structural parameters

Figure 26 and Figure 27 in Appendix 8.2 show historical decomposition of economic growth. Because the economic growth in Armenia is very volatile, it is not reasonable to explain it by permanent productivity shock (RBC without persistent productivity), but to explain by temporary one (RBC with persistent productivity). The structural change in economic growth after 2008 financial crisis is clearly visible from the second model, where permanent productivity has persistence in it.

Table 9 (see Appendix 8.2) gives the variance decomposition of economic growth for RBC models. In the model with persistent productivity, temporary productivity shock explains about 84% variation (on average) of economic growth in both short and long horizons. In the second model permanent productivity shock explains about 50% variation of economic growth.

6 Forecast performance: comparison with VAR models

This section compares out-of-sample forecast performance of the estimated RBC and DSGE models with different econometric models. To conduct out-of-sample forecast experiments, we use recursive regressions scheme. For out-of sample forecast evaluations, we divide the whole sample into two parts, particularly in-sample and out-of-sample periods. In our experiment, in-sample period includes first 60 % of observations, while out-of-sample period 40 % of observations. This means that if the whole sample includes period from 2000Q2 to 2018Q4 (75

Impulse responses, shock decomposition and other Dynare output files are available upon request.

observations), then in-sample period includes 2000Q2 to 2011Q2 (45 observations), while out-of-sample period includes observations from 2011Q3 to 2018Q4 (30 observations). The recursive simulation scheme proceeds as follows. Firstly, models are estimated using sub-sample from 2000Q2 to 2011Q2 (45 observations) and generate 1 to 12 steps-ahead forecasts. Secondly, the sample size is added by one (46 observations, 2000Q2 – 2011Q3) and generate again 1 to 12 steps-ahead forecasts. We continue increasing the sample size by one and generate 1 to 12 steps-ahead forecast until the sample size 63 (2000Q2 – 2015Q4). Then the sample size is added by one but only generate 1 to 11 steps-ahead forecasts (since there are 75 observations in total). This process continues until we have 74 observations in the sample, where the one step-ahead forecast can be computed. In such a way, it is obtained 30 one step-ahead forecasts, 29 forecasts for 2-steps-ahead, 28 for 3-steps–ahead and 19 forecasts for 12-steps-ahead. Table 5 shows RMSEs of considered models.

	AR (1)	VAR (1)	BVAR (1)	RBC with persist. prod.	RBC without persist. prod.	DSGE	DSGE with demand shock	DSGE with markup shock	DSGE with demand and markup shocks
RMSE-statistics for different forecast horizons of GDP growth									
1Q	2.62	2.82	2.86	2.60	2.58	3.04	3.07	2.68	2.57
2Q	2.72	2.77	3.34	2.60	2.59	2.59	2.62	2.58	2.61
4Q	2.63	2.63	2.48	2.58	2.60	2.75	2.72	2.84	2.69
8Q	2.66	2.69	2.62	2.59	2.60	2.58	2.59	2.55	2.60
12Q	2.66	2.67	2.68	2.59	2.60	2.64	2.65	2.60	2.63

Table 5: Out-of-sample prediction performance

As it is mentioned above, data of GDP growth, inflation and nominal interest rate are linked to DSGE models. In RBC, we link only data on GDP, because there is no role for nominal variables in these models. So AR process is taken for comparing with RBC models. For all the above mentioned models, we consider only one lag models, because it is checked by the Lag Length Criteria test that in all models the optimal number of lags is one.

Table 6 reports gains and losses of out-of-sample RMSE for RBC and DSGE models relative to VAR models.

	RBC with persist. prod. to AR	RBC with persist. prod. to VAR	RBC with persist. prod. to BVAR	RBC without persist. prod. to AR	RBC without persist. prod. to VAR	RBC without persist. prod. to BVAR	DSGE to AR	DSGE to VAR	DSGE to BVAR
	Percentage gains (+) or losses (-) relative to VAR models								
1Q	+0.76	+7.80	+9.09	+1.53	+8.51	+9.79	-16.1	-7.80	-6.29
2Q	+4.41	+6.14	+22.2	+4.78	+6.50	+22.5	+4.78	+6.50	+22.5
4Q	+1.90	+1.90	-4.03	+1.14	+1.14	-4.84	-4.56	-4.56	-10.9
8Q	+2.63	+3.72	+1.15	+2.26	+3.35	+0.76	+3.01	+4.09	+1.53
12Q	+2.63	+3.00	+3.36	+2.26	+2.62	+2.99	+0.75	+1.12	+1.49

	DSGE with demand shock to AR	DSGE with demand shock to VAR	DSGE with demand shock to BVAR	DSGE with mark. shock to AR	DSGE with mark. shock to VAR	DSGE with mark. shock to BVAR	DSGE with dem. and mark. shock to AR	DSGE with dem. and mark. shock to VAR	DSGE with dem. and mark. shock to BVAR
	Percentage gains (+) or losses (-) relative to VAR models								
1Q	-17.2	-8.87	-7.34	-2.29	+4.96	+6.29	+1.91	+8.87	+10.2
2Q	+3.68	+5.42	+21.6	+5.15	+6.86	+22.7	+4.04	+5.78	+21.8
4Q	-3.42	-3.42	-9.68	-7.98	-7.98	-14.5	-2.28	-2.28	-8.47
8Q	+2.63	+3.72	+1.15	+4.14	+5.20	+2.67	+2.26	+3.35	+0.76
12Q	+0.38	+0.75	+1.12	+2.26	+2.62	+2.99	+1.13	+1.50	+1.87

Table 6: Out-of-sample prediction performance

The out-of-sample forecast statistics show the good forecast performance of the RBC models relative to the VAR models. In the one-quarter ahead and two-quarter ahead forecast horizons RBC models and DSGE with demand and mark-up shocks show the good forecast performance relative to all the VAR models. All DSGE models forecast worse than VAR models at the four-quarter ahead horizon, RBC models does worse only relative to BVAR. At the one-quarter ahead horizon, DSGE models does considerably worse than AR models besides the model with demand and mark-up shocks. Overall, our structural models show good results in forecasting because they are simple closed economy models with assumption of balanced growth path.

7 Conclusion

This paper discusses the evaluation of structural parameters and estimated potential economic growth of Armenian economy using different specifications of DSGE and RBC models with the inclusion of balanced growth path. We estimate the models for a sample of Armenia over the period from the second quarter of 2000 to the fourth quarter of 2018 using Bayesian estimation techniques. We structure a simple closed economy DSGE model for Armenia, because it is useful for the process of linking data to the simple model. But we keep in our minds to expand the model and structure a small open economy DSGE with domestic and foreign productivity trends. For deriving the natural rate of output in our DSGE model, we construct the flexible price economy block parallel with the sticky block, which is identical to the RBC representation of the model. Looking at the estimated parameters of the DSGE models, we can record the interesting upward trend of Calvo parameter when adding demand and markup shocks both separately and together. The main driver of Calvo parameter growth is markup shock. The estimated high posterior mean value of price stickiness shows, that prices in Armenia are sticky and the volatile part of headline inflation is mostly explained by price mark-up shock. Looking at the estimation results of RBC models, there is no crucial differences between the mode values of economy's parameters, only standard errors of shocks are different. Estimated potential economic growth from DSGE model with demand and mark-up shocks is consistent with historical events of Armenian economy. Moreover, having estimated potential growth from simple RBC, its correlation with DSGE estimated growth is quite high (approximately 0.94). But having estimated identical (high correlated) DSGE potential growth with RBC's one does not mean that we have perfectly estimated potential growth. And finally, we show that our models are able to compete with VAR models in out-of-sample prediction of Armenia's economic growth. So we can use DSGE model with balanced growth path for forecasting purposes.

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8 Appendix

8.1 Appendix A. The Steady State

This appendix calculates steady states of all the endogenous variables. We use the superscript ss to denote the steady state values. The steady state values are found by using the following technique:

For example, if $Y_t = C_t + I_t$, then this equation also occurs in steady state, $Y^{ss} = C^{ss} + I^{ss}$.

We assume that $A^{ss} = 1$, $\Pi^{ss} = 1.01$ (inflation target) and $\mu_z^{ss} = 1.015$ (the average economic growth for Armenia from the second quarter of 2000 to the fourth quarter of 2018). Using A^{ss}, Π^{ss} and μ_z^{ss}, we can find the other steady state variables of the model. Writing (4.1.15) in steady state, we have the following:

$$C^{ss} = \frac{1}{\beta} C^{ss} \Pi^{ss} \big(R^{ss}\big)^{-1} \mu_z^{ss} \tag{8.1.1}$$

From (8.1.1):

$$R^{ss} = \frac{1}{\beta} \Pi^{ss} \mu_z^{ss} \tag{8.1.2}$$

Using (4.1.17), steady state of R_t^k can be represented by the following:

$$R_k^{ss} = \frac{1}{\beta} \mu_z^{ss} - (1 - \delta) \tag{8.1.3}$$

Writing (4.3.27) in steady state, it becomes:

$$\big(\Pi^{ss}\big)^{1-\varepsilon} = \theta + (1 - \theta)\big(\Pi^{*ss}\big)^{1-\varepsilon} \tag{8.1.4}$$

From (8.1.4):

$$\Pi^{*ss} = \left(\frac{\big(\Pi^{ss}\big)^{1-\varepsilon} - \theta}{1 - \theta}\right)^{\frac{1}{1-\varepsilon}} \tag{8.1.5}$$

Then using (4.3.25) the following ratio is found:

$$\frac{x_1^{ss}}{x_2^{ss}} = \frac{\varepsilon - 1}{\varepsilon} \frac{\Pi^{*ss}}{\Pi^{ss}} \tag{8.1.6}$$

From (4.3.22), we can write:

$$MC^{ss} = \frac{x_1^{ss} - \theta\beta\big(\Pi^{ss}\big)^{\varepsilon} x_1^{ss}}{Y^{ss}} \tag{8.1.7}$$

From (4.3.23) Y^{ss} can be expressed by the following:

$$Y^{ss} = x_2^{ss}\big(1 - \theta\beta(\Pi^{ss})^{\varepsilon-1}\big) \tag{8.1.8}$$

Inserting (8.1.8) into (8.1.7), the marginal cost steady state value is derived:

$$MC^{ss} = \frac{x_1^{ss}}{x_2^{ss}} \frac{1 - \theta\beta\big(\Pi^{ss}\big)^{\varepsilon}}{1 - \theta\beta(\Pi^{ss})^{\varepsilon-1}} \tag{8.1.9}$$

Already knowing MC^{ss} and R_k^{ss}, and using (4.3.10) equation, we can find the steady state value of real wage W^{ss}:

$$W^{ss} = \left[(1-\alpha)^{1-\alpha}\left(\frac{\alpha}{R_k^{ss}}\right)^{\alpha} MC^{ss}A^{ss}\right]^{\frac{1}{1-\alpha}} \tag{8.1.10}$$

From (4.1.14), I^{ss} can be represented as a function of K^{ss}:

$$I^{ss} = \frac{\mu_z^{ss} - (1-\delta)}{\mu_z^{ss}} K^{ss} \tag{8.1.11}$$

From (4.3.8):

$$\frac{K^{ss}}{N^{ss}} = \frac{\alpha}{1-\alpha}\frac{W^{ss}}{R_k^{ss}}\mu_z^{ss} \tag{8.1.12}$$

Writing (4.3.1) in steady state, it gets the form:

$$Y^{ss} = A^{ss}\left(\frac{K^{ss}}{N^{ss}}\right)^{\alpha}\frac{N^{ss}}{(\mu_z^{ss})^{\alpha}} \tag{8.1.13}$$

Inserting (8.1.11) and (8.1.13) into (4.4.3):

$$A^{ss}\left(\frac{K^{ss}}{N^{ss}}\right)^{\alpha}\frac{N^{ss}}{(\mu_z^{ss})^{\alpha}} = C^{ss} + \frac{\mu_z^{ss} - (1-\delta)}{\mu_z^{ss}} K^{ss} \tag{8.1.14}$$

Having $\frac{K^{ss}}{N^{ss}}$, we can find the following ratio:

$$\frac{C^{ss}}{N^{ss}} = A^{ss}\left(\frac{K^{ss}}{N^{ss}}\right)^{\alpha}\frac{1}{(\mu_z^{ss})^{\alpha}} - \frac{\mu_z^{ss} - (1-\delta)}{\mu_z^{ss}}\frac{K^{ss}}{N^{ss}} \tag{8.1.15}$$

(4.1.16) can be expressed by the following way:

$$W^{ss} = \left(N^{ss}\right)^{1+\phi}\frac{C^{ss}}{N^{ss}} \tag{8.1.16}$$

Rewriting (8.1.16), we find value of labor in steady state:

$$N^{ss} = \left[W^{ss}\left(\frac{C^{ss}}{N^{ss}}\right)^{-1}\right]^{\frac{1}{1+\phi}} \tag{8.1.17}$$

Then, having $\frac{C^{ss}}{N^{ss}}$ and N^{ss} , steady state consumption C^{ss} can be found as follows:

$$C^{ss} = \frac{C^{ss}}{N^{ss}}N^{ss} \tag{8.1.18}$$

We have all the necessary values to find the following endogenous variables in steady state using (8.1.13) , (8.1.11) , (4.3.22) and (4.3.23) equations:

$$Y^{ss} = A^{ss}\left(\frac{K^{ss}}{N^{ss}}\right)^{\alpha}\frac{N^{ss}}{(\mu_z^{ss})^{\alpha}} \tag{8.1.19}$$

$$K^{ss} = \frac{K^{ss}}{N^{ss}} N^{ss} \tag{8.1.20}$$

$$I^{ss} = \frac{\mu_z^{ss} - (1-\delta)}{\mu_z^{ss}} K^{ss} \tag{8.1.21}$$

$$x_1^{ss} = \frac{Y^{ss} MC^{ss}}{1 - \theta\beta(\Pi^{ss})^{\varepsilon}} \tag{8.1.22}$$

$$x_2^{ss} = \frac{Y^{ss}}{1 - \theta\beta(\Pi^{ss})^{\varepsilon-1}} \tag{8.1.23}$$

Using the same technique, all the steady state values are found in flexible price economy's block.

8.2 Appendix B. DSGE and RBC models' properties

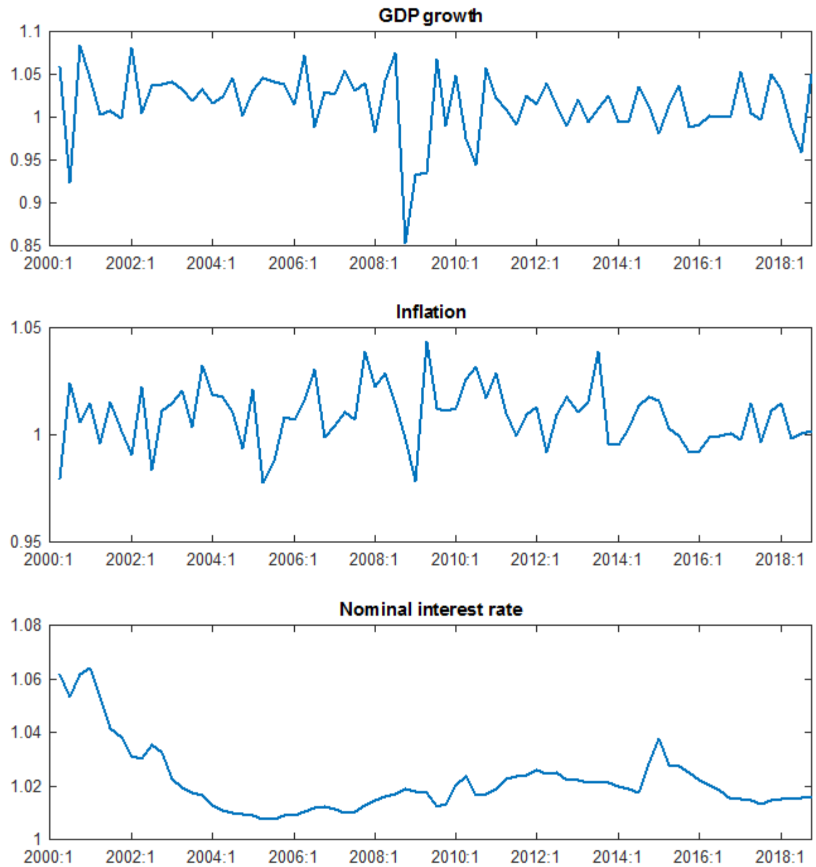

Figure 7: Gross GDP growth, Gross Inflation and Gross Nominal Interest Rate

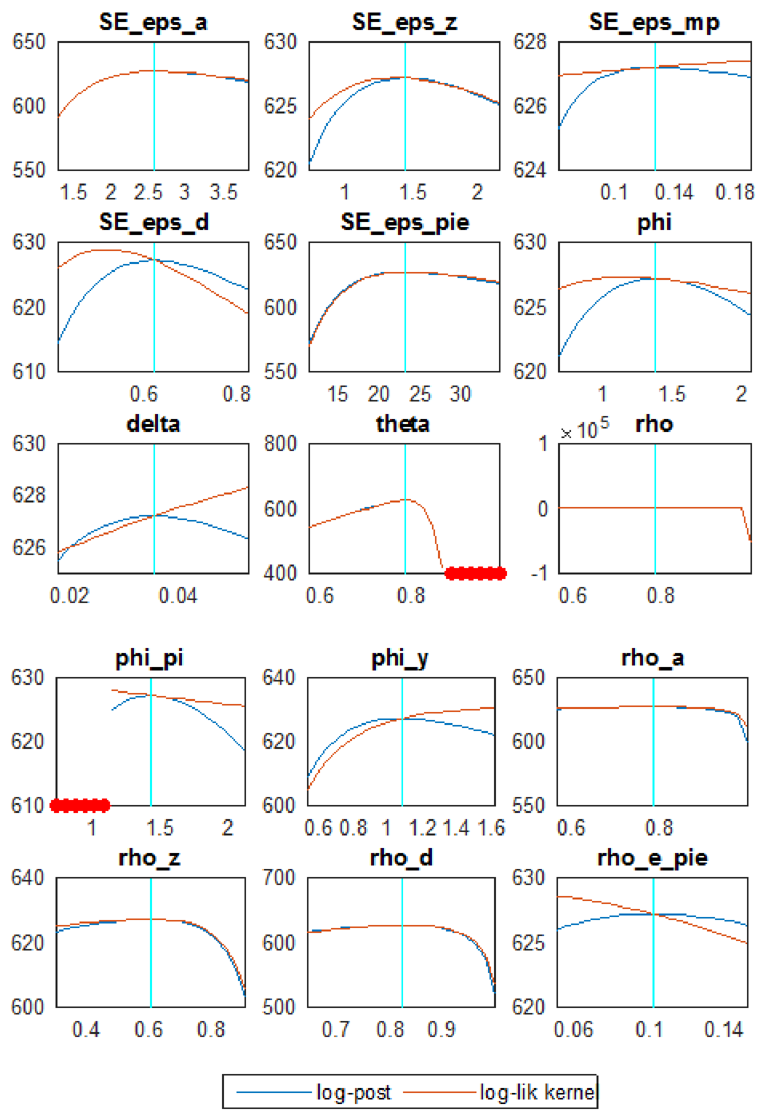

Figure 8: Mode check plots (DSGE with demand and mark-up shocks)

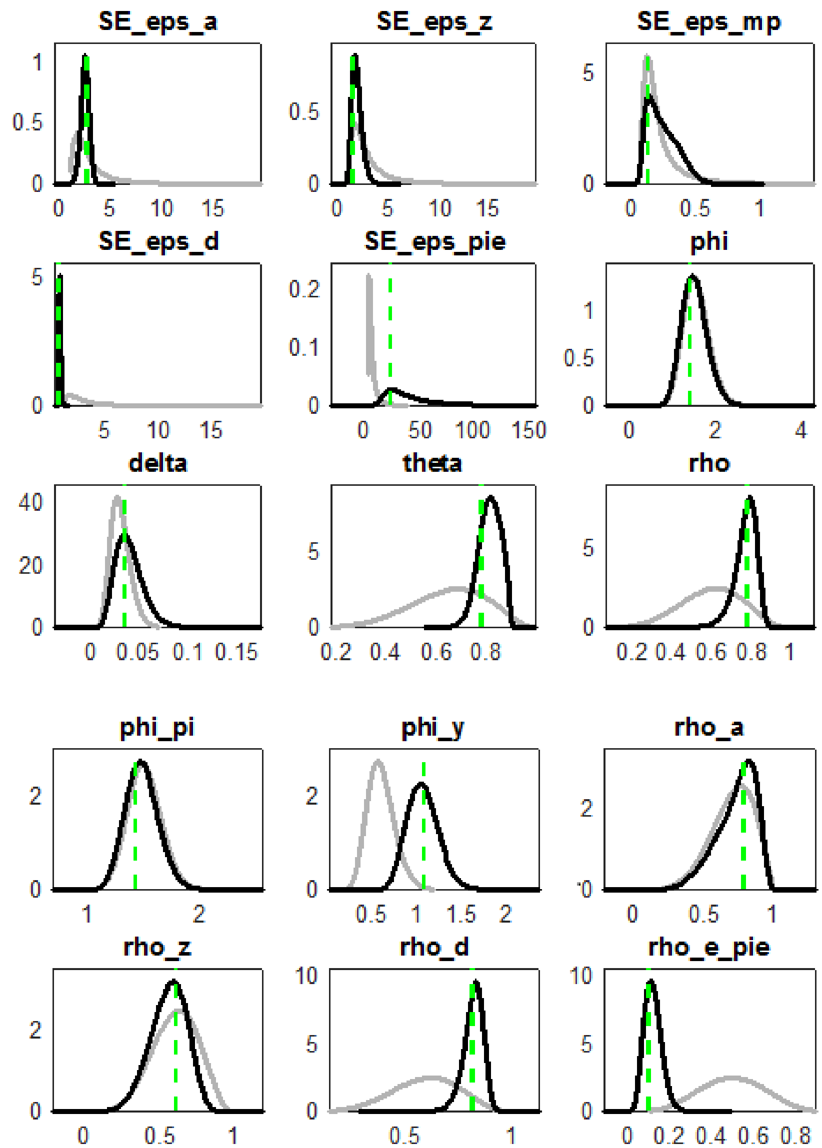

Figure 9: Prior and posterior distributions (DSGE with demand and mark-up shocks)

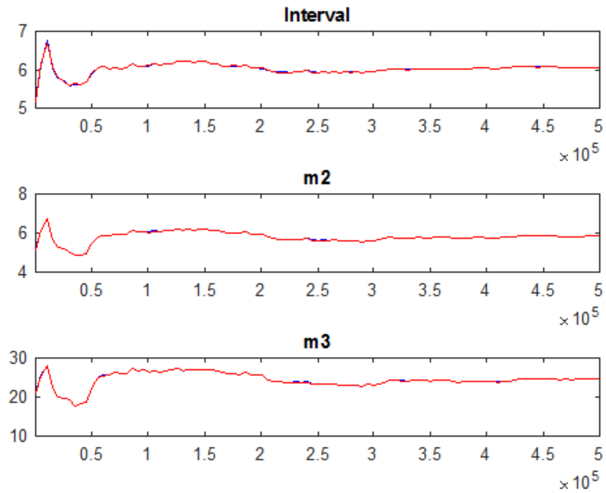

Figure 10: Multivariate convergence diagnostic (DSGE)

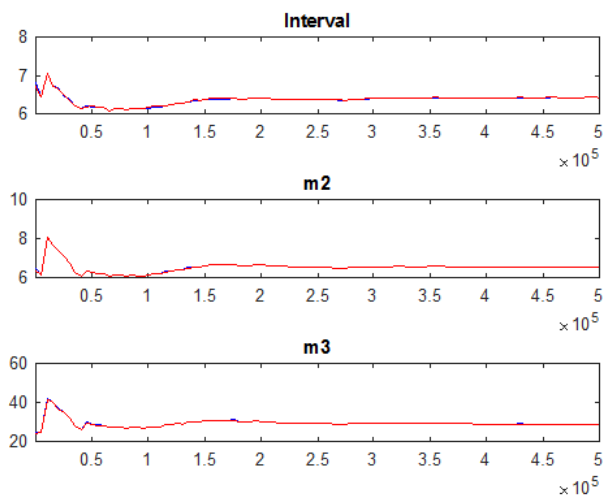

Figure 11: Multivariate convergence diagnostic (DSGE with demand shock)

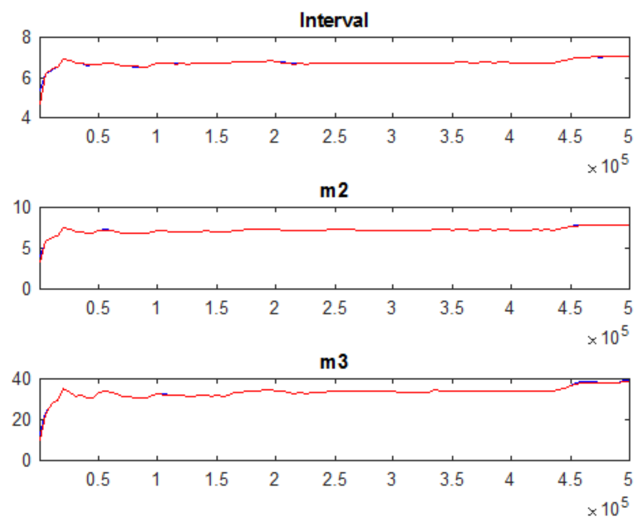

Figure 12: Multivariate convergence diagnostic (DSGE with mark-up shock)

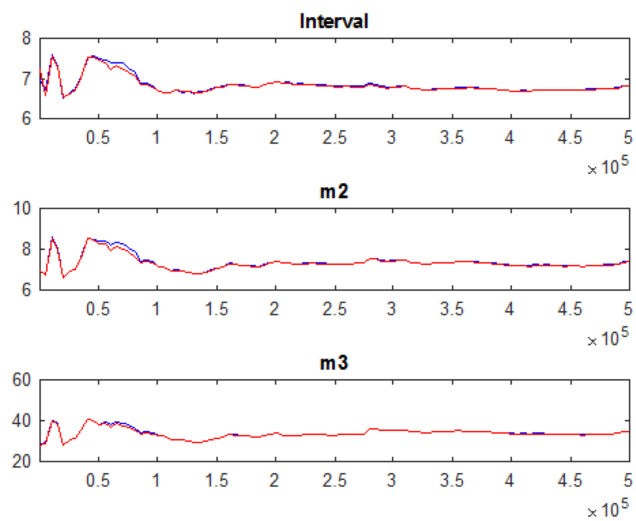

Figure 13: Multivariate convergence diagnostic (DSGE with demand and mark-up shocks)

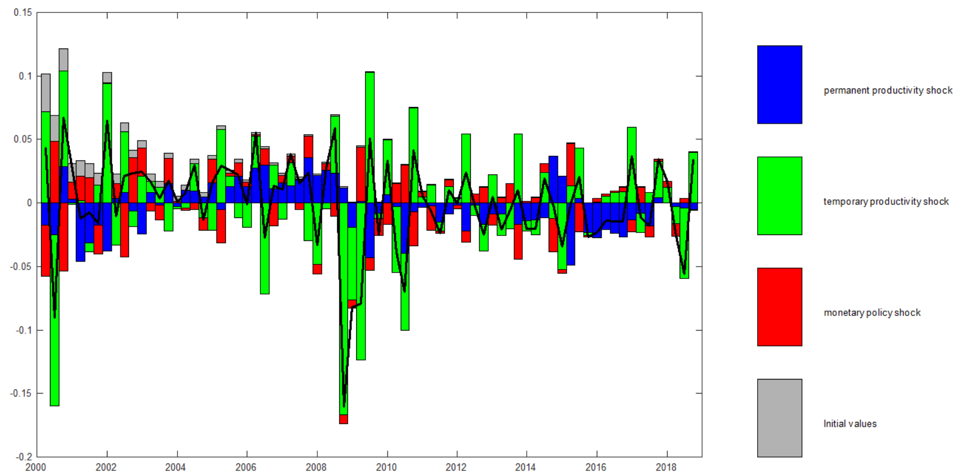

Figure 14: Historical decomposition of economic growth (DSGE)

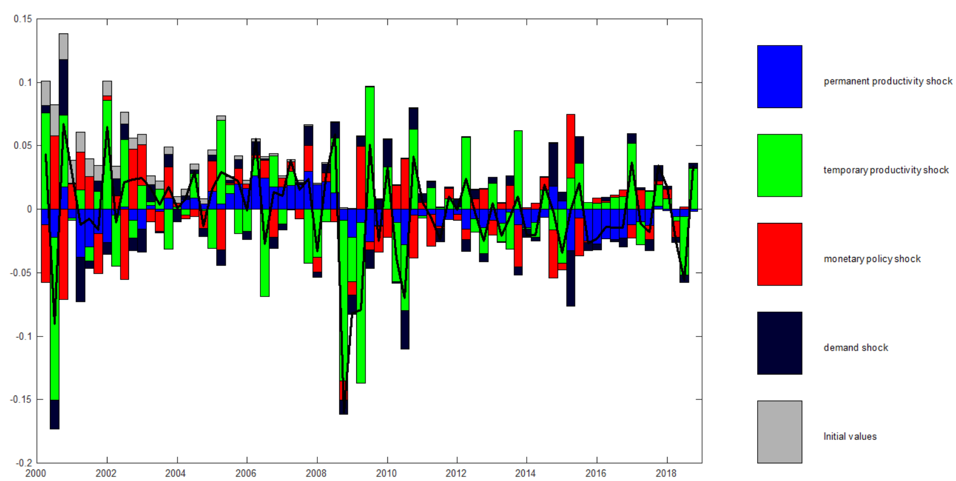

Figure 15: Historical decomposition of economic growth (DSGE with demand shock)

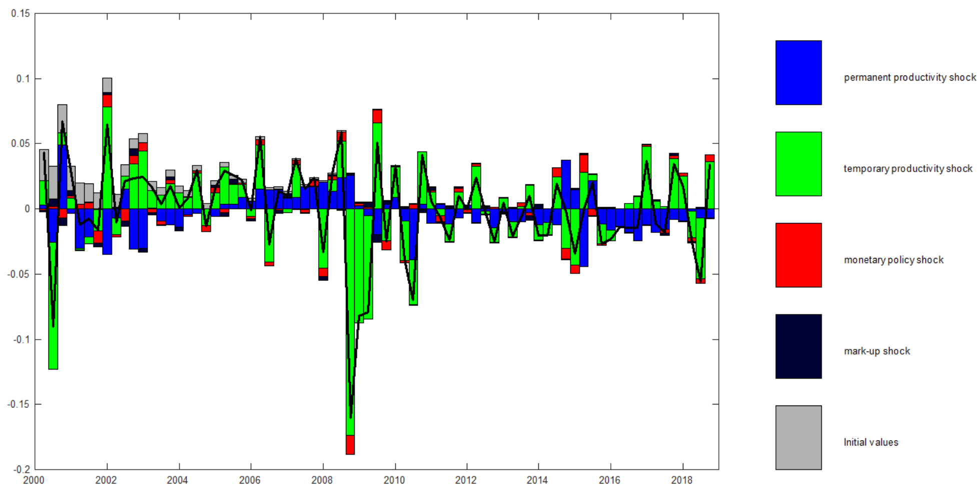

Figure 16: Historical decomposition of economic growth (DSGE with mark-up shock)

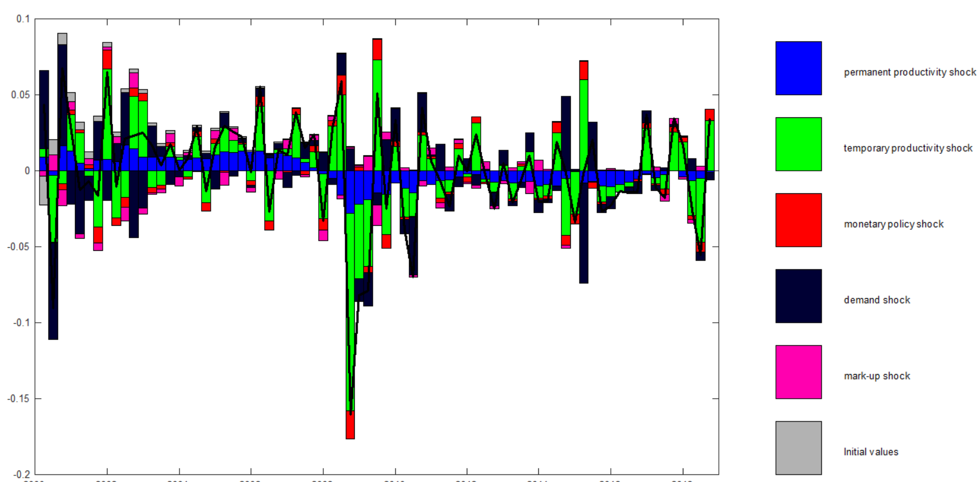

Figure 17: Historical decomposition of economic growth (DSGE with demand and mark-up shocks)

Model	Shock	Description	Economic growth				
			1Q	2Q	5Q	20Q	100Q
DSGE	ε_a	temporary productivity shock	77.07	72.19	71.83	71.95	71.94
	ε_z	permanent productivity shock	14.42	13.18	13.99	14.07	14.08
	ε_{mp}	monetary policy shock	8.51	14.63	14.18	13.99	13.98
DSGE with demand shock	ε_a	temporary productivity shock	62.79	51.55	51.61	52.04	52.06
	ε_z	permanent productivity shock	15.92	13.16	13.70	13.70	13.70
	ε_{mp}	monetary policy shock	11.63	17.90	17.58	17.36	17.35
	ε_d	demand shock	9.66	17.39	17.11	16.90	16.89
DSGE with mark-up shock	ε_a	temporary productivity shock	80.37	78.45	75.69	74.53	74.49
	ε_z	permanent productivity shock	16.55	15.58	18.84	20.15	20.21
	ε_{mp}	monetary policy shock	3.07	5.43	5.15	5.01	4.99
	ε_π	mark-up shock	0.01	0.24	0.32	0.31	0.31
DSGE with demand and mark-up shocks	ε_a	temporary productivity shock	51.80	41.39	41.79	41.94	41.93
	ε_z	permanent productivity shock	14.69	10.98	11.76	11.90	11.93
	ε_{mp}	monetary policy shock	3.53	5.12	4.96	4.94	4.93
	ε_π	mark-up shock	0.14	0.82	1.00	0.99	0.99
	ε_d	demand shock	29.84	41.69	40.49	40.23	40.22

Table 7: Conditional variance decomposition: Economic growth

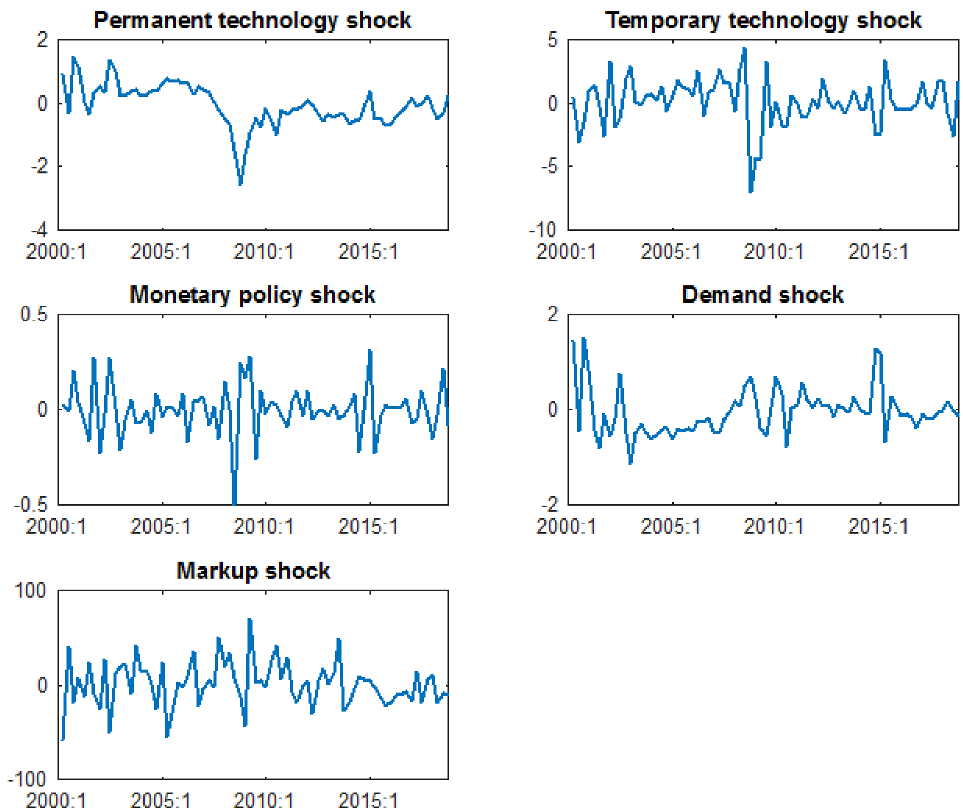

Figure 18: Historical shocks (DSGE with demand and mark-up shocks)

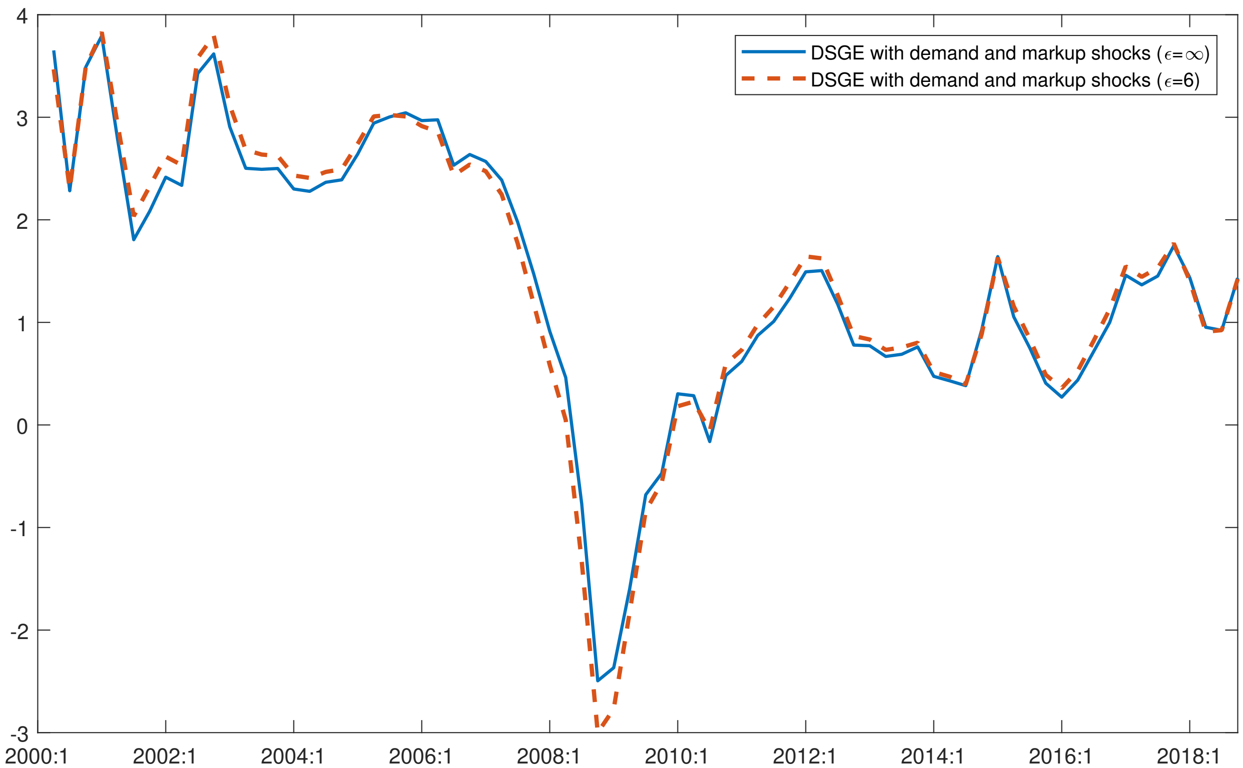

Figure 19: Estimated potential economic growth

	Description	Prior dist.	Mean	SD	RBC with persistence prod.(mode)	RBC without persistence prod.(mode)	RBC with volatile and non-volatile persistence prod.(mode)
φ	Labor supply elasticity	Gamma	1.5	0.3	1.4229	1.4285	1.4279
δ	Depreciation rate of capital	Beta	0.03	0.01	0.0269	0.0274	0.0265
α	Share of capital in production	Beta	0.4	0.12	0.3695	0.3330	0.3990
ρ_a	Persistence of temporary prod.	Beta	0.7	0.15	0.6292	0.7511	0.6338
ρ_z	Persistence of permanent prod.	Beta	0.6	0.15	0.6753	-	-
ρ_{μ_z}	Persistence of the smoothed comp. of permanent prod.	Beta	0.8	0.1	-	-	0.7969
ε_a	Temporary productivity shock	Inv. Gamma	3.5	Inf	2.3948	1.7171	2.3464
ε_z	Permanent productivity shock	Inv. Gamma	3.5	Inf	1.5929	3.7603	-
$\varepsilon_z^{'}$	Volatile component of permanent productivity shock	Inv. Gamma	3.5	Inf	-	-	1.6048
ε_{μ_z}	Non-volatile comp. of permanent productivity shock	Inv. Gamma	3.5	Inf	-	-	1.3206
	The value of Likelihood				**-143.97**	**-141.96**	**-143.46**

Table 8: The modes of structural parameters

Model	Shock	Description	Economic growth				
			1Q	2Q	5Q	20Q	100Q
RBC with persistent productivity	ε_a	temporary prod. shock	89.48	84.36	81.75	81.19	81.12
	ε_z	permanent productivity shock	10.52	15.64	18.25	18.81	18.88
RBC without persistent productivity	ε_a	temporary prod. shock	48.97	49.99	51.07	51.31	51.30
	ε_z	permanent productivity shock	51.03	50.01	48.93	48.69	48.70

Table 9: Conditional variance decomposition: Economic growth

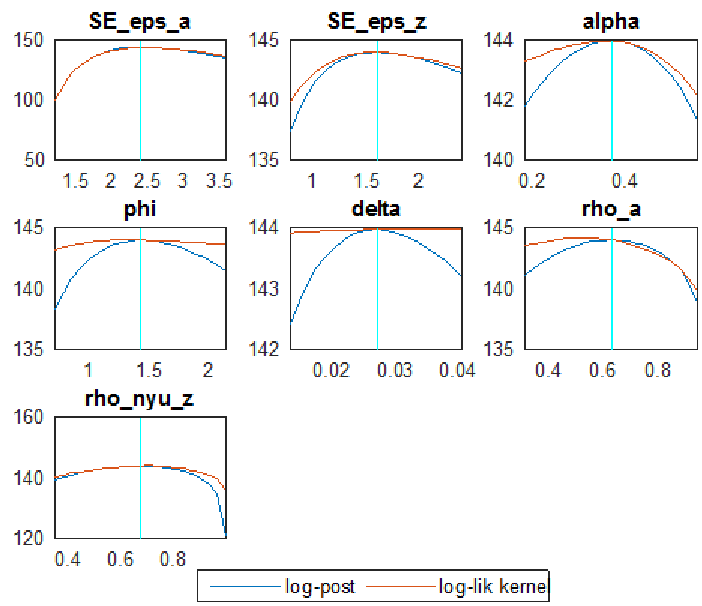

Figure 20: Mode check plots (RBC with persistent productivity)

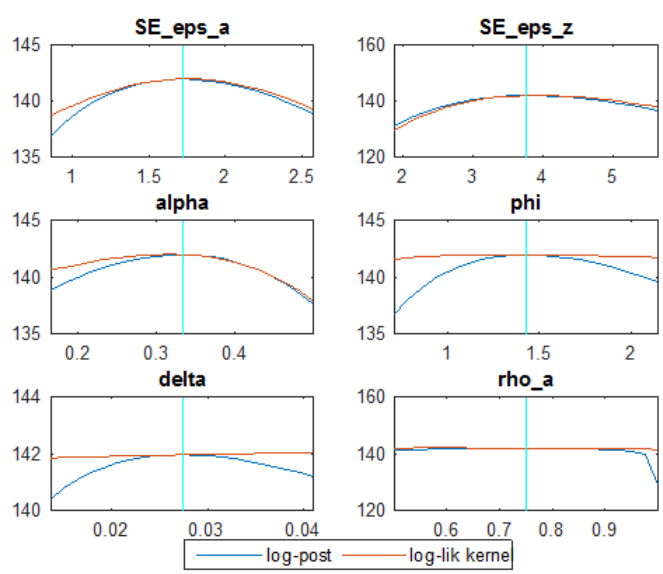

Figure 21: Mode check plots (RBC without persistent productivity)

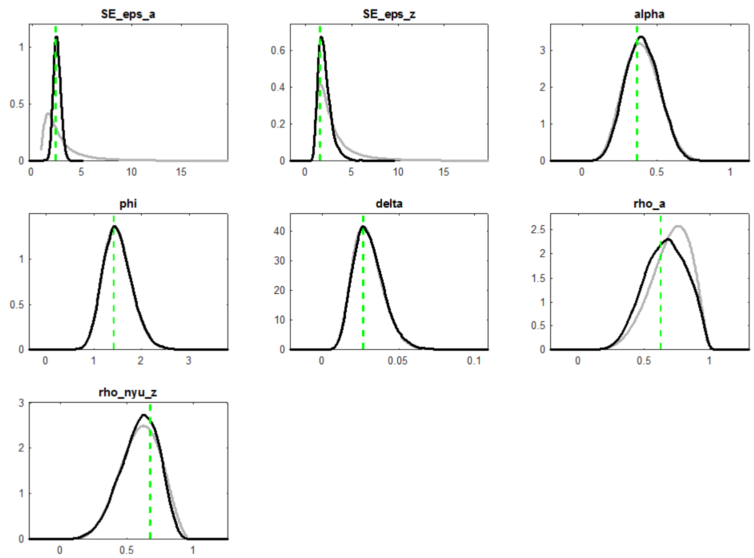

Figure 22: Prior and posterior distributions (RBC with persistent productivity)

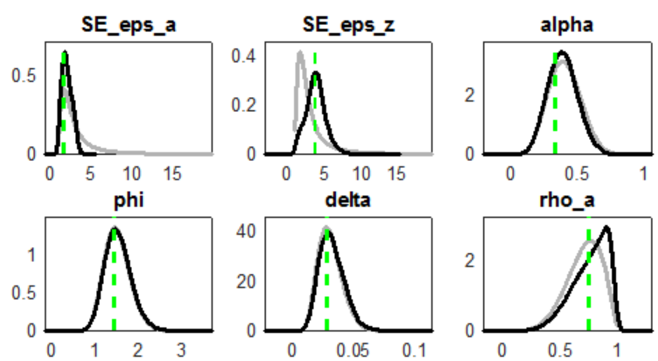

Figure 23: Prior and posterior distributions (RBC without persistent productivity)

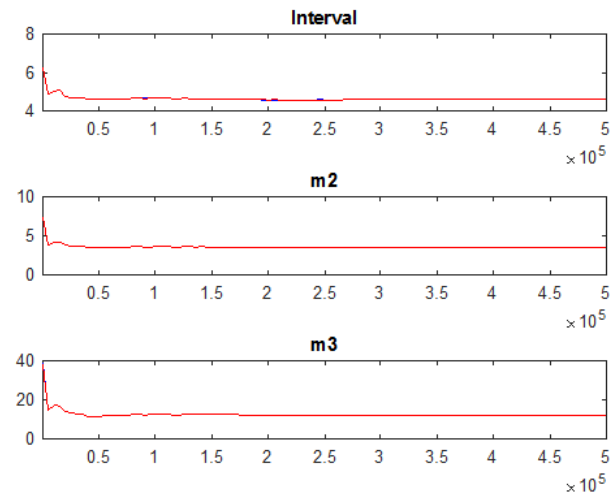

Figure 24: Multivariate convergence diagnostic (RBC with persistent productivity)

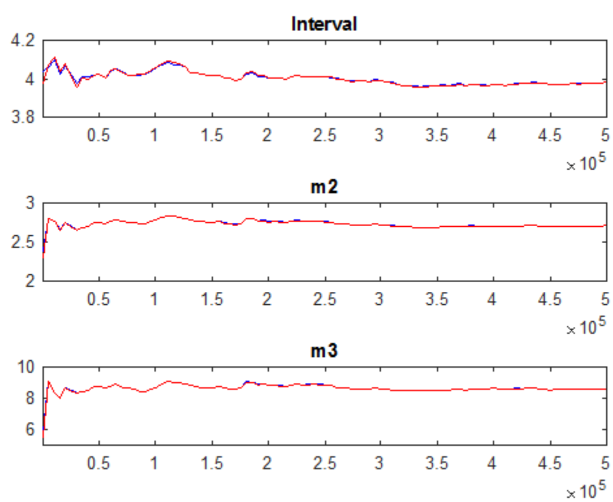

Figure 25: Multivariate convergence diagnostic (RBC without persistent productivity)

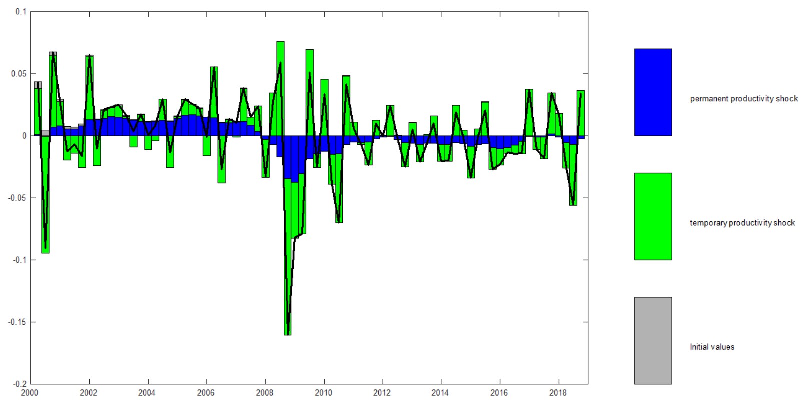

Figure 26: Historical decomposition of economic growth (RBC with persistent productivity)

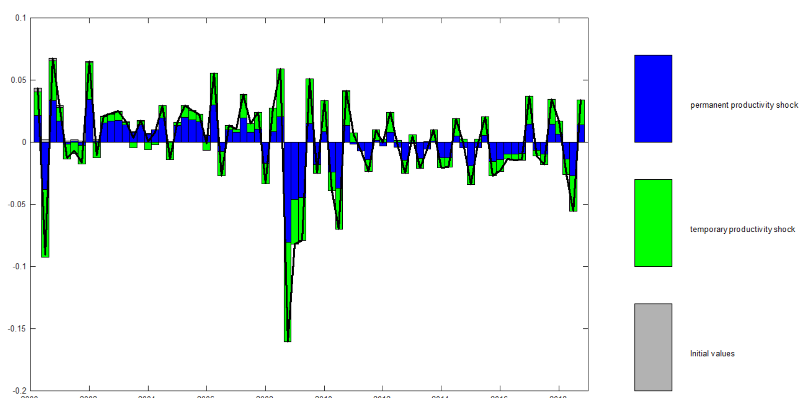

Figure 27: Historical decomposition of economic growth (RBC without persistent productivity)

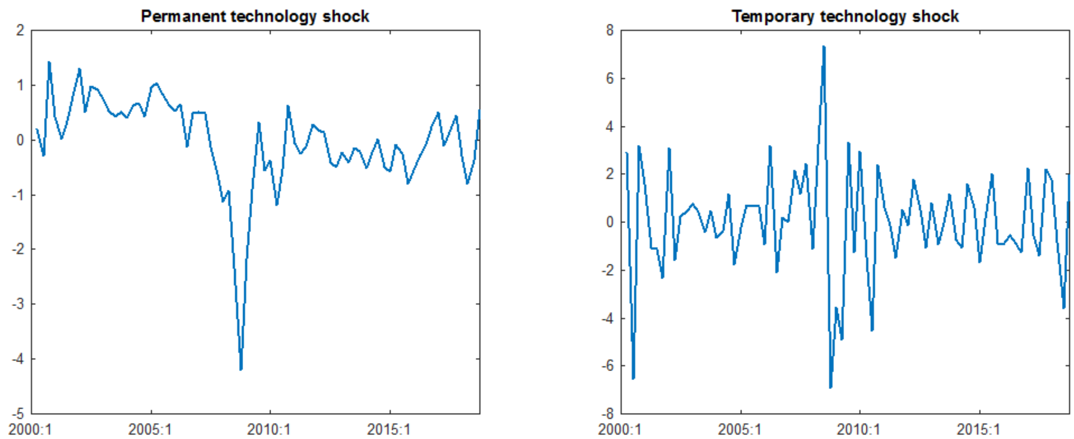

Figure 28: Historical shocks (RBC with persistent productivity)

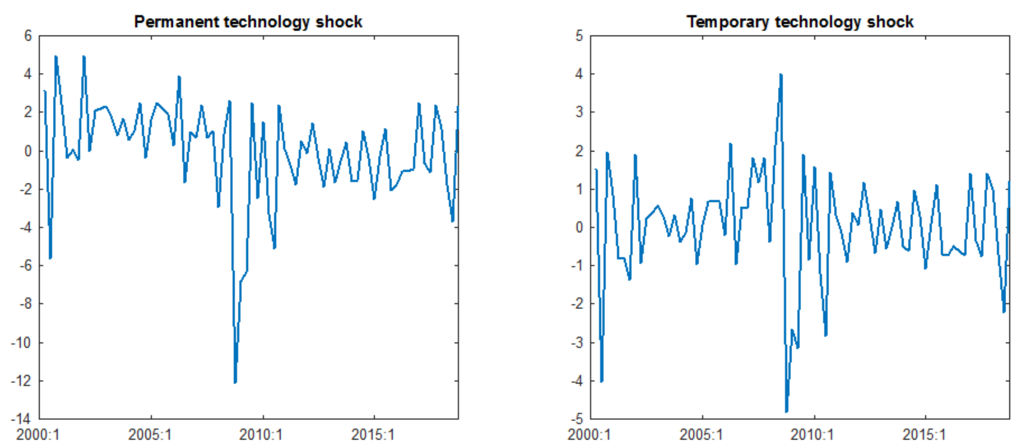

Figure 29: Historical shocks (RBC withiout persistent productivity)